

Correspondence.

CURIOUS FACTS ABOUT NUMBERS.

To the Editor of the SCIENTIFIC AMERICAN:
The theorems given in the article on "More Curious Facts About Numbers" in last week's issue of the SCIENTIFIC AMERICAN are not new, but merely special cases of Fermat's theorem. This well-known proposition is usually stated: If p is a prime number, and x is any integer, n a multiple of p , then $x^{p^n} - 1 \equiv 1 \pmod{p}$, or

$$x^{p^n} - 1 \text{ is divisible by } p. \quad (1)$$

It easily follows that for any integral value of x

$$x^p - x \text{ is divisible by } p. \quad (2)$$

For $x^p - x = x(x^{p-1} - 1)$, and either the first or the second factor of the right member is divisible by p . (Throughout these deductions p is supposed to represent a prime number.)

In regard to divisibility, the writer of the "Curious Facts about Numbers" obtained three results, viz.:

1. $x^7 - x$ is divisible by 7, and $x^{13} - x$ is divisible by 13.

2. $x^{13} - x$ is divisible by 2, 5, 7, and 13.

3. Either $x^5 + 1$ or $x^5 - 1$ is divisible by 11.

The first results represent simply two special cases of (2), viz., the cases $p=7$, and $p=13$, but (2) is true for any other prime value of p . Thus, numbers of the form $x^p - x$ can be divided by 2, those of the form $x^p - x$ by 3, $x^p - x$ by 5, etc. Or, to illustrate concretely: $2^{17} - 2$ can be divided by 17, $11^{17} - 11$ by 37, etc.

The second result can be deduced by factoring $x^{13} - x$.

$x^{13} - x = x(x^5 - 1)(x^5 + 1)$, and $x(x^5 - 1)$ is a multiple of 7, hence $x^{13} - x$ is a multiple of 7. Similarly, by considering that $x(x^4 - 1)$, $x(x^2 - 1)$, and $x(x - 1)$ are factors of the given expression, it follows that 5, 3, and 2 are divisors of $x^{13} - x$. Hence all numbers of the form $x^{13} - x$ are divisible by 2, 3, 5, 7, and 13, a more complete result than the one given by Mr. Springer. It is clear that this method may be applied to all numbers of the form $x^p - x$, since $x^p - x$ can always be resolved into factors. Thus, $x^6 - x$ may be considered a multiple of the following expressions: $x(x^5 - 1)$, $x(x^3 - 1)$, $x(x^2 - 1)$, $x(x - 1)$, and hence numbers of the form $x^6 - x$ are divisible by 61, 31, 13, 11, 7, 5, 3, and 2.

The third result is also a special case of Fermat's theorem, for according to (1) we have

$$x^{11} - 1 \text{ is divisible by } 11,$$

or $(x^5 - 1)(x^6 + 1)$ is divisible by 11, i. e., either $x^5 - 1$, or $x^6 + 1$ is divisible by 11. In general, since p is an odd number (excepting $p=2$), $p-1$ is even,

$$\frac{p-1}{2} \text{ is an integral number. Therefore}$$

$$x^{p-1} - 1 = (x^{\frac{p-1}{2}} - 1)(x^{\frac{p-1}{2}} + 1).$$

Hence, according to (1) $(x^{\frac{p-1}{2}} - 1)(x^{\frac{p-1}{2}} + 1)$ is divisible by p , and since p is prime, either $x^{\frac{p-1}{2}} - 1$, or $x^{\frac{p-1}{2}} + 1$ is divisible by p . Thus $x^6 \pm 1$ is divisible by 13, $x^{14} \pm 1$ is divisible by 29, etc.

Finally it may be said that the formulæ for integral values of a , b , and c , satisfying the equation $a^2 + b^2 = c^2$ are very old, and quite generally known. They can be easily obtained by the general methods of solving indeterminate equations of the second degree.

ARTHUR SCHULTZE.

New York University, November 25, 1908.

A DEFENSE OF THE WRIGHT SYSTEM OF PROPELLERS.

To the Editor of the SCIENTIFIC AMERICAN:
I have read from time to time criticisms of various details of the Wright machine, particularly as to the use of twin propellers. The unfortunate accident at Fort Myer has in most cases been used as a strong argument against them.

It strikes me that it is about time that someone had something to say in defense of this feature. I was personally a witness of the accident and fully believe that the real immediate cause of the accident was the breaking of the rear rudder and its gear.

To be sure, this was caused by one of the propellers striking a guy-wire, which held the top strut in place; but it is extremely probable that if a single propeller or tandem propellers had been in use the resultant injury to the rear rudder would have been the same if a rear rudder guy had projected in the path of the single propeller. To understand how this injury to the rear rudder caused the accident it is well to consider just how the warping of the planes in conjunction with the rear rudder is used to maintain the transverse stability and also to make turns.

If the rear of the right wing is depressed a certain amount, the rear of the left wing raised a corresponding amount, and the plane forced straight forward, then, as the angle of incidence of the right wing is increased and that of the left wing diminished, the right side of the plane will tend to rise. However, when this is done (i. e., the wing warped) the head resistance of both planes is increased a certain amount, and if we consider the planes alone and leave out of the question the forward movement, it will be seen that, under the circumstances, the planes will tend to turn to the right under the resistance of the air and the force of gravity. If we move our rear rudder to steer the planes to the left, then we can overcome the tendency to move to the right caused by the warping of the planes. In this case the right side of the plane will be tilted up, if the plane is moving through still air. Or this movement can be used to counteract a tendency to overturn the planes to the right caused by a strong gust of wind coming from the left. In turning to the left the rudder is used, and the planes are tilted so as to incline the machine to the inside of the curve in a similar manner to that in which a bicyclist inclines his wheel in rounding a curve.

My theory of the accident is as follows: Most of

the turns during all the flights of Orville Wright were made to the left. This of course would tend to stretch the left-hand rudder stays. The accident happened just as a turn was being made or about completed. It is probable that Mr. Wright was about straightening up for a straight run. To do this he would need to steer to the right, which would slacken the left rudder guy and cause it to sag in the path of the left propeller with disastrous results, both the propeller and the rudder being put out of commission. For a time the right-hand propeller continued to turn, and this tended to tilt and steer the machine still further to the left.

Naturally, even after the power was turned off, the response to the warping of the planes was sluggish, and the machine lost headway owing to the increased head resistance caused by the warping. The result was to cause it to pitch forward, by reason of the change of the center of pressure caused by the loss of forward motion. Before the longitudinal balance could be regained the machine struck the ground.

An examination of any of the pictures of the machine after the accident will show the broken rear rudder. As all witnesses seem to agree on the fact that the machine struck the ground head on a very cursory examination of the pictures will convince any thinking person that the damage to the rear rudder could not have been caused by the machine striking the ground at that end.

The slight mishap to Wilbur Wright in which one of his chains broke goes to prove that the loss of the propelling effect of one of the propellers is not in itself enough to cause a serious accident, since he easily came to the ground without any damage to the machine or passenger. In fact the turning effect was probably much stronger in his case than in that of Orville Wright, since there was part of the left propeller blade in action which would tend to counteract that of the other.

Twin screws have certain advantages on boats, and these are very much accentuated on aeroplanes. In the first place there is with single screws a tendency to tip the plane sidewise in the opposite direction in which the screw turns, which effect is entirely neutralized with twin screws.

Furthermore a screw shows much more efficiency at low than at high speeds. The practical limit of the diameter of the screws is about the distance between the planes. Hence by using two screws instead of one, the thrust will be doubled simply by doubling the power. The real lesson to be learned from the accident is not that twin propellers must be discarded, but that braces on any type of airship must be so arranged that it is impossible for them to come in contact with the blades of the screws. Santos Dumont learned this very early in his experiments with dirigibles.

One correspondent criticized the use of a chain drive and advocated the use of bevel gears. It is probable that no one realizes more than the Wrights themselves that their machine has many shortcomings in minor details. The fact must be borne in mind that the Wrights were not persons of unlimited means, and naturally they chose the methods which were the least expensive and likely to give the results wanted. It is probable that the chain drive as used by them costs less than a tenth of what even a passably good bevel drive would have cost and gives service that could only be surpassed by a bevel drive of the very best design, workmanship, and material.

The Wright machines of to-day are but copies of a successful experimental machine and as such naturally lack many of the minor refinements which are bound to come when the machine becomes a regular manufactured article. However, even in its present form it would seem to be capable of winning most of the prizes offered for various feats of aviation.

HAROLD S. BROWN.

Boston, Mass., December, 1908.

The Current Supplement.

To many a man who has had to do with electric currents in some form or other, the question has risen, either in his own mind while at work, or in some discussion with a friend: "What does direct current mean? What is the difference between a direct current and an alternating current?" Mr. S. A. Fletcher states the difference very simply and clearly in the opening article of the current SUPPLEMENT, No. 1721. One of the features of the Dayton meeting of the Ohio Society of Mechanical, Electrical, and Steam Engineers was a discussion of the relative merits of the steam and gas engine. That discussion is summarized. Italian naval architects have suggested the use of concrete as an armor for warships. What it costs to break an Atlantic steamship record is set forth. G. H. Bryan gives a very succinct account of aeronautic principles. Dr. Andrew Wilson writes on the human engine, in which he carries out the idea that a good many analogies exist between machines of man's making and his own body. Concrete is admirably adapted for many purposes upon the modern country estate. It may be successfully used by the laborer with fair intelligence under proper supervision. Mr. Linn White in a very exhaustive article gives carefully worked-out details of the manner in which material may be thus used. An interesting article describes two remarkable sense organs, one of which is a thermoscopic eye, and the other a light-projecting eye.

At Bolthead, on the Devonshire coast, a wireless station has just been opened by the postmaster-general of the British post office. This station is intended to establish communication with ships at sea. It is stated that this is the first of a series of similar stations which are to be maintained by the post office throughout Great Britain.

grades prepared for the purpose of curing fish, meats, etc. "Coarse solar" includes all coarse salt made by solar evaporation. "Rock" salt includes all salt mined and shipped without special preparation. "Mill" salt is that used in gold and silver mills, and "other grades" includes all low-grade or No. 2 salt used in salting cattle and for fertilizers, track purposes, etc. "Brine" includes all salt liquor used in the manufacture of soda ash, sodium bicarbonate, sodium hydrate (caustic soda), and other sodium salts or brine sold without being evaporated to dryness.

Production of salt by grades in the United States 1907, in barrels:

Table and dairy salt.....	3,537,157
Common fine salt.....	7,684,638
Common coarse salt.....	2,055,054
Packers salt.....	422,324
Solar salt.....	862,929
Rock salt.....	5,809,328
Other grades.....	110,227
Brine.....	9,222,471
Total production, barrels.....	29,704,128
Value.....	\$7,439,551

In 1894 salt was placed on the free list and importations increased to 434,155,708 pounds in 1894 and to 520,411,822 pounds in 1896. In 1897 salt was again made dutiable, and salt in bags, barrels, or other packages is subject to a duty of 12 cents per 100 pounds (33.6 cents per barrel) and salt in bulk is taxed 8 cents per 100 pounds (22.4 cents per barrel). The duty on imported salt in bond used in curing fish taken by licensed vessels engaged in fishing and in curing fish on the navigable waters of the United States or on salt used in curing meats for export may be remitted.

The imports came from the United Kingdom, Italy, British West Indies, and Spain, named in the order of importance. From these four sources over 90 per cent of both quantity and value of the imports were derived.

The exports of salt of domestic production from the United States in 1907 was 61,603,422 pounds, valued at \$232,195. Most of this salt went to Cuba, Canada, Mexico, and Panama.

In the following table the statistics of salt production in the principal salt-producing countries of the world in 1906 are shown as far as statistics are available. The production of Turkey is not included. The industry in that country, as in Austria-Hungary, is a government monopoly, with no statistics of production published. No statistics are available from Russia since 1903.

World's Production in Short Tons.

	Quantity.	Value.
1906. United States.....	3,944,133	\$6,658,350
1906. United Kingdom.....	2,201,293	2,900,983
1906. France.....	1,496,923	4,198,329
1906. German Empire.....	2,059,096	5,000,823
1904. Japan.....	773,776	4,852,049
1906. Italy.....	586,424	1,119,786
1906. Austria.....	414,465	9,717,164

Our graphical illustrations really explain themselves. Thus our upper engraving shows all the salt of the oceans thrown up on the land and sea, it would cover the entire earth to a depth of 112 feet or well above the roof of the Capitol at Washington. The next comparison shows the per capita consumption of the Frenchman 9 pounds, the Englishman 13 pounds, and the American 11 pounds. Then follow the two cones of salt, that in the sea 4,800,000 cubic miles and 325,000 cubic miles for salt on the land. Little wonder that Mont Blanc appears as a mere speck. The last comparison is the yearly production of salt in the United States, which shows a tidy little barrel 700 feet high and 500 feet in diameter at its widest point. Truly the small condiment of our table presents an enormous mass in the aggregate.

In the rebuilding of the Quebec Bridge, it is said that the engineers who have been retained by the Dominion government will consider the advisability of providing for at least ten feet more headroom from the water than existed under the former structure. It may be remembered that the height of the old Quebec Bridge was 150 feet above high water, and that the Montreal Board of Trade feared that this would prevent the large ships of the future from passing up the river to Montreal. The height advocated by the Montreal Board of Trade was 190 feet, which, however, can only be obtained at a cost which is regarded as prohibitory. The tallest masts now arriving in Montreal are those of the Allan liner "Virginian," which are of a height of 141 feet. Under the old Quebec Bridge these would have passed with nine feet to spare. But the masts of the "Empress of Britain" and the "Empress of Ireland," of the Canadian Pacific line, are 154 feet high, and for these it would have been necessary to await the ebb of the tide if they wished to pass under.