MODELS FOR ILLUSTRATING THE STRAIN ON STRUCTURES.

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There is hardly any doubt that the value of mathematical formulæ to constructing engineers is frequently overrated. In fact, even in connection with the most complicated construction of a statically undetermined nature a figure calculated on a given basis is frequently considered as an absolute gage of safety.

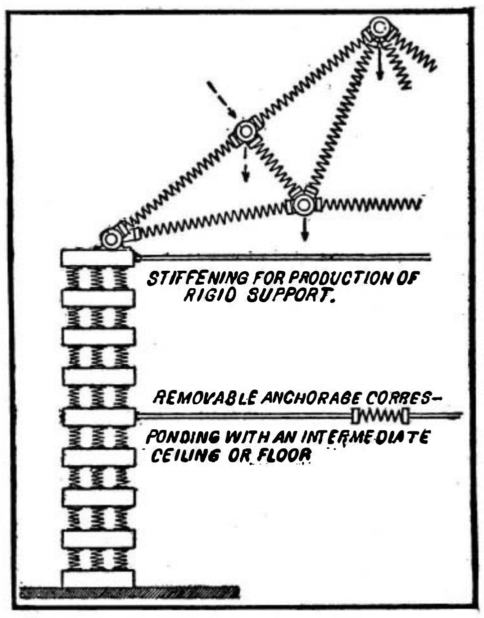

Fig. 6.—Showing interrelation of strains on various parts of a structure.

Even if all factors determining the distribution of strains in a given structure could be reduced to a mathematically available form, it should be considered that statical calculations are possible only for a given condition of the structure, being liable to alteration as a consequence of any elastic or non-elastic change in the shape of some part of the structure, or of its abutments.

The distribution of strains in an arch free from any joints may, for instance, very well be calculated mathematically with a practically sufficient degree of accuracy for a given load, if the abutment be invariably rigid, or if the displacement or distortion be known. However, there are in reality no invariably rigid abutments, while the displacement and torsion generally depend on so many factors, that mathematical calculations fail to determine them. Starting from the results of a statical calculation for a given condition, any possible departures from the hypotheses made, and their possible influence on the distribution and magnitude of actual strains, should accordingly be gaged in each case.

The influence of these factors should by no means be underrated, being sometimes so considerable, that the results of calculation afford an entirely incorrect representation of the conditions of safety of a structure. Another striking instance is that of framed structures in which the rigidity of webs is liable to entirely impair the results of mathematical calculation, in connection with which hinged connections are generally pre-supposed. In order, therefore, to supply an independent check on mathematical instruction, based on practical experience, Mr. E. Carlipp of Erlangen, Germany, has designed an ingenious device for illustrating the distribution of strains in structures, of which we herewith give a short illustrated description, as well as some photographs.

Any structure under the action of forces (provided that no motion of the whole takes place as a consequence of a rupture in equilibrium) is liable to undergo some alteration in shape, which at each point bears a certain ratio to the strain obtaining there. If this alteration in shape be known, the distribution of strains will thus be likewise given.

The models used by Carlipp are made up alternately of springs and rigid sections (see Fig. 1) in order to allow the actual alterations in shape and strain to be visually observed. Any kind of forces (such as pull, thrust, inflection, torsion, rotation) will produce in a composite body of this kind, alterations in shape, which allow the nature and relative magnitudes of strains to be recognized. The strength of the springs corresponds to the coefficient of elasticity of the material; and by fitting springs of variable strength, the distribution of strains on a structure, made of different materials is readily illustrated.

The strains on the springs f_1 (in Fig. 1), for instance, will be the smaller the less the springs f_2 are able to yield, that is the greater the amount of load they are able to deal with. The same instance also corresponds to cases in which the coefficient of elasticity in regard to thrust is greater than elasticity in regard to pull. The points of maximum strain in many cases play an essentially important part.

Fig. 7.—Diagram of arch with sliding abutment.

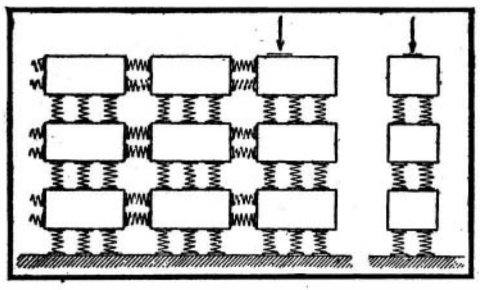

Fig. 8.—Distribution of loads inside a structure.

If the model be lined with paper, plaster, etc. (Fig. 2), the fissures resulting from given conditions of load and given arrangements of abutments will allow their location to be determined, while illustrating the relative amounts of strain in a specially striking manner, corresponding to real life. In the case of framework structures, the framework rods are generally formed by simple continuous springs connected by joints in the connections. These joints may be fixed by clamps with a view to illustrating the influence of rigidity of the webs. The various elements can be combined in many different ways, so as to obtain the most varied models suitable for instruction. Though all possible cases cannot obviously be enumerated in the present article, some instances will be quoted, illustrating the possibilities of these mod-

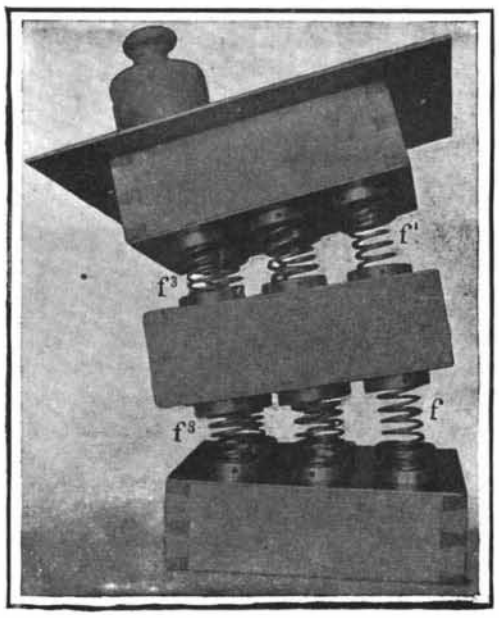

Fig. 1.—Composite model of springs and rigid sections.

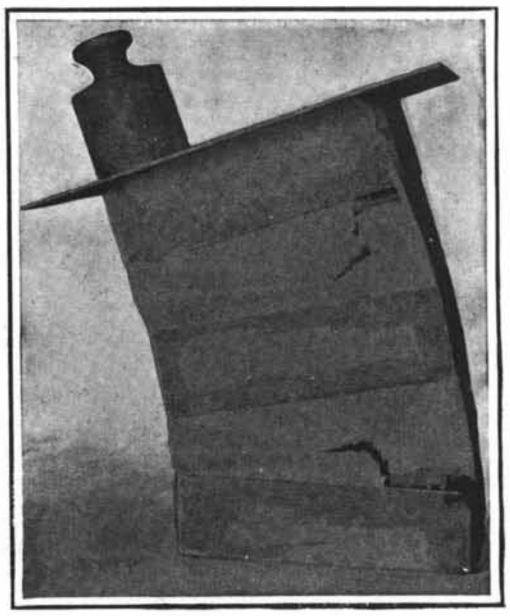

Fig. 2. Model showing strains by the fissures in the paper covering.

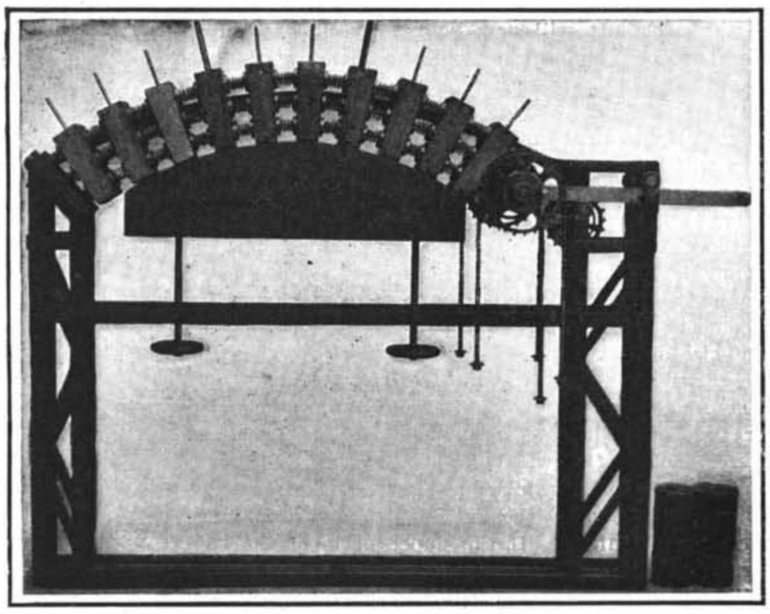

Fig. 8.—Arch supported while being loaded.

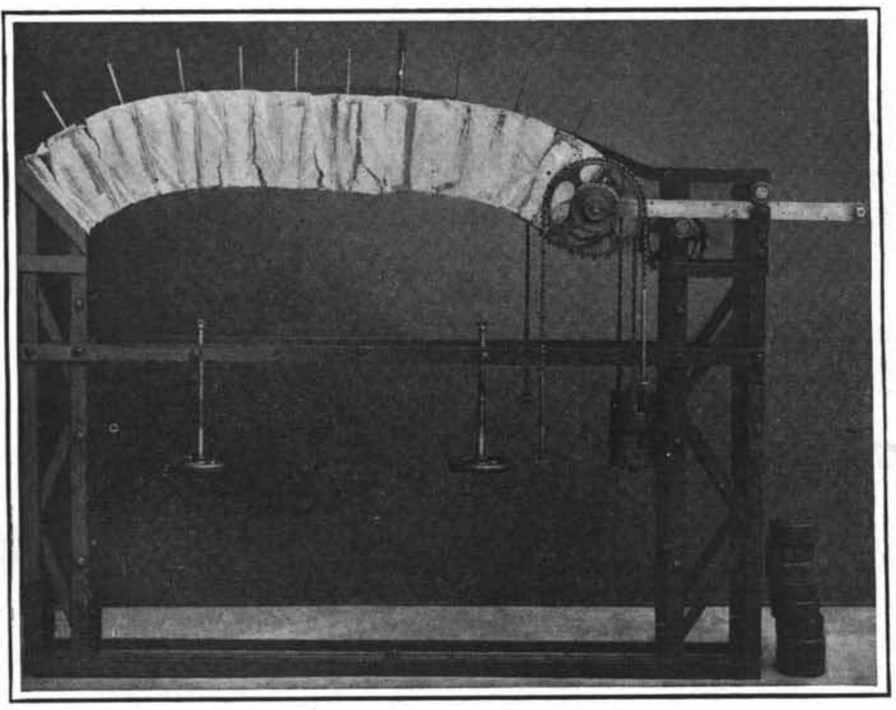

Fig. 4.—Arch under load, showing breaks at crown and abutments.

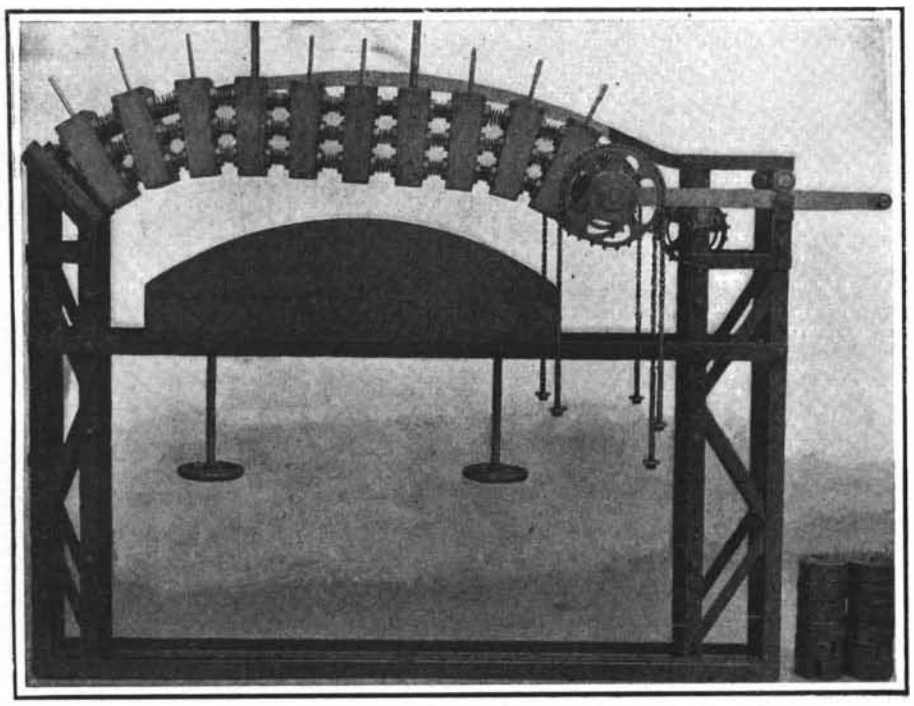

Fig. 5.—Arch with sliding abutment at one end.

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els, both for demonstration before an auditorium and for self-instruction.

Fig. 5 represents a model in which the distribution of strains in an arch is illustrated for different loads, and with different abutments, one of the abutments being free to rotate round an axle and to move in a horizontal direction. This horizontal motion can be counteracted by the weight *H* (Fig. 7), and the rotation by the weight *G;* the former thus representing the horizontal thrust and *G* the torque of the abutment. Both the horizontal displacement and rotation can be completely eliminated by arresting the abutment with the aid of suitable weights. Demonstration with this model could, for instance, be performed in the following manner:

The arch is first supported by a simple center liable to be lowered, while being loaded either by its own weight alone, or by an additional effective load of approximately uniform distribution (Fig. 3), thus illustrating the fact that all springs are by no means compressed uniformly, a lined arch being liable under certain conditions even to show some fissures; though with the actual shape and load of the arch there may be ascertained a curve of pressures which practically coincides with the central line of the arch.

It may even be shown that any moments of deflection obtaining in the arch (that is, any departures of the line of pressure from the central line) are mainly due to the elastic shortening of the arch, these moments being the greater as the ratio of the thickness of the arch to the pitch is greater, any arch in connection with which this ratio does not assume too small a value, being in reality something intermediary between an arch and an imbedded girder, and the girder action being the more predominant as the shape of the arch approximates that of a girder. If now a sufficient amount of weight be added in *H*, to prevent any displacement of the abutment, the stop being loosened, *H* will correspond to the horizontal thrust after an elastic elongation of the arch in the case of rigid abutments. By shortening the arch, an identical effect to an increase in the distance of abutments will be brought about. By increasing *H*, the abutments can be made to approach to one or other, the elastic shortening of the arch being compensated and uniform strains being produced in all the sections of the arch. It is also seen that the horizontal thrust decreases with the elastic elongation of the arch. It will now be readily understood in what manner the influence of the yielding of abutments can be illustrated: Each given weight *H* and *G*, the load remaining the same, will correspond to a given displacement or distortion, and the latter in turn will influence in a given manner the distribution of strains in the arch. In the case of a lined arch (Fig. 4) fissures can be produced, with a given load, at the crown, at will, either on the top or below, by simply increasing or decreasing *H*. The influence of the most different loads can likewise be shown for the most varied arrangements of abutments, and the influence of transverse forces is readily illustrated by the relative displacement of the rigid parts.

The model represented in Fig. 6 is mainly intended for illustrating the fact that a structure, from the statical point of view, always exerts its effects as a whole, the actual strain on any part of the structure being dependent on those to which the remaining parts are submitted. If the upper parts of the supports (walls) be solidly reinforced, the roof structure will rest on abutments, capable of only relative displacement, and according as the joints are hinged or clamped fast, a given distribution of strains will be obtained with a given load. The distribution of strains will be susceptible to alteration both in the main truss of a roof and in the supports, if the stiffening of the abutments being withdrawn, the elastic alterations in the shape of the supports are allowed to make themselves felt.

Another model designed by Mr. Carlipp is represented in Fig. 8. When calculating a structure, the section of which has sides of different length (walls, bridges, girders, etc.), the load being approximately concentrated, incorrect results are obtained if the sections are supposed to remain plane. The model represented in Fig. 8 now illustrates with a given drop of loads, the distribution of strains inside the structure and the pressure exerted on the soil.

Girders, etc., in which the effect of transverse forces is manifested separately, that is to say, by a vertical as well as by a horizontal mutual displacement of the rigid parts, can be combined in a similar manner to that illustrated in Fig. 8.

Even in cases which are not at all susceptible to calculation, such as that of plates resting on non-uniform abutments (e. g., triple abutments) such a model will allow the distribution of forces to be ascertained for any load.

Simple girders can be constructed in a manner quite similar to the arch in Figs. 5 and 7, being demonstrated for the most various arrangements of abutments (isolated on two abutments wholly or partly imbedded, continually resting on supports of equal or unequal height) and under any conditions of load.

While all the possible cases cannot obviously be demonstrated in the case of technical instruction, even a limited number of demonstrations will doubtless fit students to acquire an appropriate conception of the phenomena of elasticity obtaining in connection with real buildings, instead of being content with mechanical calculation. Teachers will be able to use such demonstrations as a basis of general instruction of a kind which is apt to be neglected in the ordinary theoretically mathematical courses, while even advanced students will avail themselves of certain models to complete their knowledge of the statical conditions of structures. It will even be possible to construct models immediately imitating a given structure to be erected, so as to afford an absolutely correct gage of the actual distribution of strains.

In order to illustrate the distribution of stresses in homogeneous bodies, all the springs should obviously be of the same thickness, being submitted to the same strain. In order to allow this result to be readily obtained, the springs are connected with the rigid parts, by a simple device, allowing the springs to be tightened or loosened, or even exchanged at a moment's notice.

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MAGNITUDE OF THE SALT INDUSTRY.

Sodium chloride or common salt is one of the most useful substances in the world and one of the most necessary to the human economy. Salt was first used as an aliment at the period of transition from the nomadic pastoral life to a more sedentary and agricultural life. Salt puts in motion the secretion of the stomach and furnishes it with some of its constituent parts. The material for the chlorine compounds of the gastric juice comes primarily from the salt of our food.

In any attempt to compute the relative abundance of the chemical elements we must bear in mind the limitations of our experience. Our knowledge of terrestrial matter extends but a short distance below the surface of the earth, and beyond that we can only indulge in speculation. The atmosphere, the ocean, and a thin shell of solids are, speaking broadly, all that we can examine. For the first two layers our information is reasonably good, and their masses are approximately determined; but for the last one we must assume some arbitrary limit. The real thickness of the lithosphere need not be considered; but it seems probable that to a depth of 10 miles below sea level the rocky material can not vary greatly from the volcanic outflows which we recognize at the surface. This thickness of 10 miles, then, represents known matter, and gives us a quantitative basis for study. A shell only 6 miles thick would barely clear the lowest deeps of the ocean.

The volume of the 10-mile rocky crust, including the mean elevation of the continents above the sea, is 1,633,000,000 cubic miles, and to this material we may assign a mean density not lower than 2.5 nor much higher than 2.7. The volume of the ocean is put at 302,000,000 cubic miles, and Prof. Frank Wigglesworth Clarke in his valuable "Data of Geochemistry," says he has given it a density of 1.03, which is a trifle too high. The mass of the atmosphere, so far as it can be determined, is equivalent to that of 1,268,000 cubic miles of water, the unit of density. Combining these data, we get the following expression for the composition of the known matter of our globe:

Density of crust		2.5	2.7
Atmosphere	per cent	0.03	0.03
Ocean	per cent	7.08	6.58
Solid crust	per cent	92.89	93.39
		100.00	100.00

In short, we can regard the surface layer of the earth, to a depth of 10 miles, as consisting very nearly of 93 per cent solid and 7 per cent liquid matter, treating the atmosphere as a small correction to be applied when needed. The figure thus assigned to the ocean is probably a little too high, but its adoption makes an allowance for the fresh waters of the globe, which are too small in amount to be estimable directly. Their insignificance may be inferred from the fact that a section of the 10-mile crust having the surface area of the United States represents only about 1.5 per cent of the entire mass of matter under consideration. A quantity of water equivalent to 1 per cent of the ocean, or 0.07 per cent of the matter now under consideration, would cover all the land areas of the globe to the depth of 290 feet. Even the mass of Lake Superior thus becomes a negligible quantity.

The composition of the ocean is easily determined from the data given by Dittmar in the report of the "Challenger" expedition. The maximum salinity observed by him amounted to 37.37 grammes of salts in a kilogramme of water, and by taking this figure instead of a lower average value we can allow for saline masses inclosed within the solid crust of the earth, and which would not otherwise appear in the final estimates. Combining this datum with Dittmar's figures for the average composition of the oceanic salts, we get the second of the subjoined columns. Other elements contained in sea water, but only in minute traces, need not be considered here. No one of them could reach 0.001 per cent.

Composition of oceanic salts.	
NaCl	77.76
$MgCl_2$	10.88
$MgSO_4$	4.74
$CaSO_4$	3.60
K_2SO_4	2.46
$MgBr_2$	.22
$CaCO_3$	.34
	100.00

Composition of ocean.	
O	85.79
H	10.67
Cl	2.07
Na	1.14
Mg	.14
Ca	.05
K	.04
S	.09
Br	.008
C	.002
	100.00

It is worth while at this point to consider how large a mass of matter these oceanic salts represent. The average salinity of the ocean is not far from 3.5 per cent; its mean density is 1.027, and its volume is 302,000,000 cubic miles. The specific gravity of the salts, as nearly as can be computed, is 2.25. From these data it can be shown that the volume of the saline matter in the ocean is a little more than 4,800,000 cubic miles, or enough to cover the entire surface of the United States, excluding Alaska, 1.6 miles deep. In the face of these figures, the beds of rock salt at Stassfurt and elsewhere, which seem so enormous at close range, become absolutely trivial. The allowance made for them by using the maximum salinity of the ocean instead of the average is more than sufficient, for it gives them a total volume of 325,000 cubic miles. That is, the data used for computing the average composition of the ocean and its average significance as a part of all terrestrial matter are maxima, and therefore tend to compensate for the omission of factors which could not well be estimated directly.

The facts that we can estimate, with a fair approach to exactness, the absolute amount of sodium in the sea, and that it is added in a presumably constant manner without serious losses, have led to various attempts toward using its quantity in geological statistics. The sodium of the ocean seems to offer us a quantitative datum from which we can reason. That is, if all the sodium in the sea were derived from the decomposition of igneous rocks, a shell of the latter one-third of a mile thick would supply the entire amount. An allowance for the sodium retained by the sedimentaries would increase this estimate to half a mile, which is the largest amount possible. All conceivable corrections tend to diminish the figure. A stratum of igneous rock, one-half mile thick and completely enveloping the globe, would furnish all the sodium of the ocean and the sediments. Joly, by a similar process of reasoning and in part from the same data, has sought to compute the geological age of the earth since erosion commenced. From Murray's estimate concerning the discharge of rivers Joly determines that 157,267,544 tons of sodium are annually poured into the sea. At this rate denudation of the land would require a period of from ninety to one hundred millions of years in order to make up the oceanic quantity of sodium. By applying certain corrections the figure is reduced to eighty or ninety millions of years as the time which has elapsed since water condensed upon the globe and aqueous denudation began.

It is not necessary to enter into the details of these and other similar calculations, for they can only be regarded as tentative and preliminary. They do, however, indicate certain possibilities and show how desirable it is that we should increase the accuracy of our data. When we know more precisely what chemical work is being done by the rivers, with annual averages for all of the greater continental streams, we may have materials for something like a fair measure of geological time. Our present knowledge on this subject is too incomplete and too unsatisfactory.

In 1907 the quantity of salt produced in the United States was 29,704,128 barrels of 280 pounds, valued at $7,439,551, says W. C. Phalen, expert of the Geological Survey.

For convenience salt is classified according to the grades by which it is sold by the producer, the grades being determined by the amount of refining, the methods employed in refining, and the purposes for which the salt is used. These grades are: "Table and dairy," "common fine," "common coarse," "packers," "solar," "rock," "milling," "brine," and "other grades." The "table and dairy" salt includes extra fine and fancy grades prepared for family use, and all grades artificially dried, used for butter and cheese making and such special brands. Under "common fine" salt are included all other grades of fine salt of first quality not artificially dried, such as those known to the trade as "C. F.," "No. 1 F," "anthracite," etc. "Common coarse" salt includes all grades coarser than "common fine" made by artificial heat, such as "steam coarse," "No. 1 coarse," "pan solar," "G. A.," "Liverpool ground," "C. C.," etc. By "packers" salt is meant those