The entire right-hand side of this equation contains 13 as a factor except

$$z^{13} - z.$$

Let us consider the possible values of this expression.

For $z = 1$,	$z^{13} - z =$	0
$z = 2$,	$z^{13} - z =$	8,190
$z = 3$,	$z^{13} - z =$	1,594,320
$z = 4$,	$z^{13} - z =$	67,108,860
$z = 5$,	$z^{13} - z =$	1,220,703,120
$z = 6$,	$z^{13} - z =$	13,060,694,010
$z = 7$,	$z^{13} - z =$	96,889,010,400
$z = 8$,	$z^{13} - z =$	549,755,813,880
$z = 9$,	$z^{13} - z =$	2,541,865,828,320
$z = 10$,	$z^{13} - z =$	9,999,999,999,990
$z = 11$,	$z^{13} - z =$	34,522,712,143,920
$z = 12$,	$z^{13} - z =$	106,993,205,379,060

Examining the column of figures at the right hand, it will be seen that each and every number is divisible by 13. Consequently, the expression $x^{13} - x$ is divisible by 13. We have yet to show that it is also divisible by 7. We represent x now by $7m + n$, where n may be any number from 1 to 6. The expression $x^{13} - x$ now becomes

$$x^{13} - x = (7m + n)^{13} - (7m + n)$$

Just as before, every term on the right-hand side, when expanded, contains the factor 7 except the expression

$$n^{13} - n$$

To examine this for values from 1 to 6, is simply to look over the column of figures last obtained beginning with the first and extending through the sixth, to see whether 7 is a factor. Upon examination, we find that it is. So that as a result of our investigation we find that all numbers of the form $x^{13} - x$ are divisible both by 7 and by 13—consequently by 7×13, or 91.

But we have not reached the end as yet. In raising numbers to the several powers, we find that powers of numbers ending in 1 also end in 1. Thus, any power of 31 will terminate in the figure 1. If a number ends in 2, the final figure of its various powers runs through the cycle 2, 4, 8, 6. Constructing a table, we find the following results:

Final Figures.	Powers.					
	1	2	3	4	5	6
1	1	1	1	1	1	1
2	2	4	8	6	2	4
3	3	9	7	1	3	9
4	4	6	4	6	4	6
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	9	3	1	7	9
8	8	4	2	6	8	4
9	9	1	9	1	9	1
0	0	0	0	0	0	0

Thus for numbers ending in 7, the final figures of the powers run through the cycle 7, 9, 3, 1. Upon examining the table closely, it will be seen that these cycles consist of one, two, or four numbers. In consequence of this periodicity, the thirteenth power of any number will terminate in the same figure as the first power, all the cycles starting afresh at this power (as also at the fifth and ninth). Consequently, if we subtract the first power from the thirteenth we shall obtain a number whose final figure is 0—the final figures of minuend and subtrahend being the same. Consequently, all numbers of the form $x^{13} - x$ terminate in 0, and are for this reason divisible by both 2 and 5. We learn, then, that all numbers of the form $x^{13} - x$ are divisible at once by 2, 5, 7, and 13—that is, by 910.

The Current Supplement.

The opening article of the current SUPPLEMENT, No. 1716, deals with the subject of the short-distance transportation of the products of the mine, field, and forest from the outlying districts to the railroad. The use of dry air in the blast furnace as patented by Gayley is described fully. Various conjectures have been made as to the cause of the destruction of Count Zeppelin's airship. One very plausible explanation ascribes the catastrophe to the effect of atmospheric electricity. The electrical principles involved are described in the SUPPLEMENT. Whether as a toy, as a scientific instrument, or as an object of practical utility, the gyroscope is of great interest. As an illustration of its possible development, a gyroscope invented by Prof. Ach of Berlin is described, which has for its object a means of securing the invariability of the axis of rotation of the gyroscope, no matter what arbitrary movements may take place in the carrier of the gyroscope. The invention comprises means to do away with the influence of gravity on the spinning body. Sir William Ramsay discusses the subject, "Do the Radioactive Gases Belong to the Argon Series?" The making of artificial diamonds is described. Mr. Alexander A. Chenoweth writes on reinforced concrete as a substitute for wooden piles, cross-ties, or sleepers. Photographic bird's eye views have hitherto been obtained by means of balloons, or kites. Dr. Neubronner has discovered that they may also be taken with the aid of carrier pigeons. The necessary apparatus is described and illustrated.

A $500 Prize for a Simple Explanation of the Fourth Dimension.

A friend of the SCIENTIFIC AMERICAN, who desires to remain unknown, has paid into the hands of the publishers the sum of $500, which is to be awarded as a prize for the best non-mathematical explanation of the Fourth Dimension, the object being to set forth in an essay the meaning of the term so that the ordinary lay reader can understand it.

Competitors for the prize must comply with the following conditions:

1. No essay must be longer than 2,500 words.

2. The essays must be written as simply, lucidly, and non-technically as possible.

3. Each essay must be typewritten and identified with a pseudonym. The essay must be inclosed in a plain sealed envelope, bearing only the pseudonym. With the essay should be sent a second plain sealed envelope, also labeled with the pseudonym, and containing the name and address of the competitor. Both these envelopes should be sent to "Fourth Dimension Editor, SCIENTIFIC AMERICAN, 361 Broadway, New York, N. Y."

4. All essays must be in the office of the SCIENTIFIC AMERICAN by April 1, 1909.

5. The Editor of the SCIENTIFIC AMERICAN will retain the small sealed envelope containing the address of the competitor and forward the essays to a Board of Judges, who will select the prize-winning essay.

6. As soon as the Board of Judges have agreed upon the winning essay, they will notify the Editor, who will open the envelope bearing the proper pseudonym and containing the competitor's true name. The competitor will be notified by the Editor that he has won the prize, and his essay will be published in the SCIENTIFIC AMERICAN.

7. The Editor reserves the right to publish in the columns of the SCIENTIFIC AMERICAN or the SCIENTIFIC AMERICAN SUPPLEMENT three or four of the more meritorious essays, which in the opinion of the judges are worthy of honorable mention.

The Editor of the SCIENTIFIC AMERICAN is now engaged in selecting the judges who will award the prize. They will be three in number, and all will be eminent American mathematicians. The names of the judges will be announced in a later issue of this journal.

Substitute for Chromate of Silver.

Running short of Lord Kelvin's glass tubes for deep-sea soundings, and finding the prices charged in America—40 cents each—rather high, Mr. John Martin of the S. S. "Akershus" informs us that he tried to find some substitute for chromate of silver, and succeeded. He bought a ¼-inch (inside diameter) water-gage glass of the same length as Lord Kelvin's tubes, closed one end with a flat piece of cork, a small rag over it and some sealing wax. Next he cut a sheet of drawing paper into narrow strips at a bookbinder's. Along the middle of the paper strip he drew a line with a copying pencil. The paper slip was then shoved up the glass tube, forming a blank scale. The lower end of the strip was turned around the edge of the tube and tied with a piece of cotton thread to prevent the paper from being displaced by the water as it rose in the tube. The tube was used in the same way as Lord Kelvin's. The depth was read off from the boxwood scale. The ink pencil line distinctly indicated how far the water reached. The guard tube containing the gage glass was made from an old condenser tube.

The Death of William Edward Ayrton.

William Edward Ayrton, the distinguished English electrical engineer and inventor, died in London on November 8 in his sixty-first year. He was educated at University College, London, and after graduation entered the Indian Government Telegraph Service. In 1873 he was appointed to the chair of natural philosophy and telegraphy at the Imperial College of Engineering, Japan. Here he remained for five years. In 1884 he was made professor of electrical engineering at the Central Technical College, South Kensington. He was president of the Physical Society, 1891-2, and president of the Institution of Electrical Engineers in 1892.

The Death of John Henry Mills.

John Henry Mills, a pioneer heating engineer and inventor of heating boilers, of Boston, Mass., died at his home in Faneuil, a suburb of that city, on Tuesday, October 6, in his seventy-fifth year. He was an authority on heating, and his work, "Heat—Its Application to the Warming and Ventilation of Buildings," is one of the standard books on the subject.

Correspondence.

USEFUL ARTIFICIAL WATERWAYS.

To the Editor of the SCIENTIFIC AMERICAN:

The proposition to retain water at the heads of large rivers and their tributaries is surely well worth careful consideration, as it will aid in retaining moisture, not only on the surface, but to great depths, replenishing old-time underground rivers and furnishing water for wells in the surrounding country. Such reservoirs can be used to keep machinery running during the dry season too, thus adding millions of dollars valuation to power plants, by steadying business, in regulating production so as to secure equal results throughout the whole year.

Yet still all the water that can be retained will prove to be an infinitesimal quantity in checking the overflow of the lower streams of any great system of rivers, such as the Mississippi and the Amazon.

The best possible command of the Mississippi River can be secured by dredging a deep and broad channel for several hundred miles inland at a proper distance from the river channel on both sides of the river.

The channel on the west side should extend from Atchafalaya Bay, keeping west of the extremely troublesome Bayou Lafourche and passing east of Opelousas and Alexandria, La., east of Pine Bluff, Ark., on to the head of White River, across the line of Missouri, to a point opposite Charleston near the Mississippi River, and Van Buren on the Iron Mountain Railway. This will drain large tracts of wet, unproductive flat lands, which will be of excellent quality and most of them fertile and productive when properly drained. There are hundreds of thousands of acres of flat, wet land along the proposed course of this west side canal.

The east side channel should begin at Bay St. Louis, extending north, crossing the Pearl River, to a point above Lake Pontchartrain. There bending west should cross the Illinois Central Railway, near the town of Amite, reaching the Mississippi Valley a few miles below Natchez, on into the Yazoo Valley, then on up that valley, passing near and east of Memphis and on to the Ohio River Valley. There bending east follow the course of that river as far as the land formation makes it desirable. At certain points it may be necessary to leave off one side of a river and take up construction on the other side. This being a question of easy decision for any competent engineer.

These channels will have a threefold purpose of usefulness: First, as channels of drainage, they will add several hundreds of millions of dollars value to the country traversed, by providing several outlets for drainage of swamps and farm land in general. For such an outlet there is a burning necessity which should have been provided for long years ago.

Second, for relieving the strain upon the levees of the Mississippi and the other rivers and bayous included in the districts traversed by the canals, also to prevent overflow by letting surplus water into these channels, through intersecting canals a any point of greatest strain. This can be done at any point where cross-sectional channels have been constructed.

Third, if set free to the traffic of all who own suitable crafts, the value for purposes of navigation alone will eventually pay the cost of construction and fat dividends besides.

These main channels can be constructed a liberal width in the beginning and shallow, to be deepened by dredging until, if found desirable, seagoing vessels of the highest tonnage can pass each other with ease, and take their loads inland to the head of these canals.

On the east side of the Mississippi River, beginning about 30 miles above Cairo and extending up to St. Louis, is an extensive bottom-land country, which would be tripled in value by a canal of sufficient capacity to carry off the local surplus water, and aid in relieving the Mississippi whenever a threatened overflow at that part of its course created a call for an extra outlet.

Between Omaha, Neb., and St. Joseph, Mo., a good deep and broad canal would aid greatly in preventing the destructive overflowing and changing of channel of the Missouri River.

At Kansas City, too, relief from inundations of the lower portions of that city can be secured by one or two canals of capacity requisite for drawing off the surplus water whenever a surplus approaches with all its threatened consequences.

On the middle and upper Ohio, along the rivers of Kansas, wherever there is a necessity in localities mentioned or others not named; wherever are found large rivers or threatening streams subject to extraordinary floods, in any part of the world, this proposed plan will bring security from ordinary ensuing disastrous consequences. Think for a little how much this means.

From the northwest corner of Lake Pontchartrain a cypress swamp extends north for many miles. This swamp has water nearly all over its surface, for the greatest part of the year. If drained, it would soon become farm land of great value on account of its fertility. This can all be drained by one canal and a levee along the shore of the lake, then pump the water over the levee into the lake.

Many other similar localities can be reclaimed in like manner. The above proposed system is of vast and far-reaching importance, and should be carefully investigated at the earliest possible opportunity afforded to those upon whom the responsibility devolves, and whose duty it is to have this work of altogether unprecedented importance performed.

Whose duty should it be, if not the unbounded duty of the people, all the people, of the great republic, the United States of America? Every citizen of this whole country should be interested. This work will benefit all of them, both individually and collectively.

J. A. STOCKFORD.

Spokane, Wash., November 2, 1908.

From the returns compiled by Lloyd's Register, it appears that, excluding warships, there were 319 vessels of 733,378 tons gross under construction in the United Kingdom at the close of September, 1908.