tions, but abstractions of things not only non-existent in nature, but which cannot possibly have independent existence.

In explanation, let us make a plain statement of the case, and we will begin with the definition of the limits of a body, or its *surface*, the limits of a surface, or *lines*, and, lastly, the limits or ends of a line, or *points*. Euclid proceeds in the reverse way, and speaks first of a point having neither length, breadth, nor thickness; then of a line having only length, and neither breadth nor thickness; and, lastly, of a surface having only length and breadth,and no thickness. The conclusion to which any one with a philosophical and critical turn of mind must arrive is that, these things being impossibilities, and having no material existence, a science based on such conceptions must have a very weak foundation; such a critic would be justified in his opinion.

The point, line and superficies, as defined by Euclid in this abstract way, can have no existence; and if geometry were really based on these principles, the science, renowned as the most positive of all positive sciences, would in reality be based on abstractions, mere notions concerning impossible things. No wonder, then, that these definitions of Euclid have been the points of attack aimed at by all those who have attempted to bring mathematics down to the level of the uncertain and unprofitable speculations of metaphysics, such persons assuming that mathematics is based on definitions of point, line, and superficies, which are absurdities in themselves.

These faulty definitions can be entirely corrected by following the suggestion made in the beginning of this article. We therefore begin with "Definition 1. The body. All bodies occupy a certain limited space, and, whether large or small, have three dimensions, length, breadth, and thickness." This is illustrated by a cube, parallelopiped, etc., and the science of physics investigates the properties of bodies (such as weight, color, hardness, etc.), and that of chemistry its component elements (such as carbon, hydrogen, oxygen, etc.); but in geometry we only consider the dimensions above given. "Definition 2. The surface. The limit of such a body is called its surface, and from this it follows that such a surface possesses length and breadth, but can have no thickness, as, by attempting to measure this, we necessarily would go either inside the body or outside of it." This is illustrated by placing a metal cube in water, and remarking that the limits between the metal and the water, where they touch, and where there is neither water nor metal, constitute the mathematical idea of a surface. "Definition 3. The line. The limit of such a surface where two sides of a body meet (its edges) is called a line; this line is common to both surfaces; and it possesses only length, and neither breadth nor thickness." This is again illustrated by a cube or pyramid, and we remark that, by attempting to measure the thickness of the edges, we necessarily would abandon one of the planes and move into the other. "Definition 4. The point. Where two or more such edges of a body meet, or the position whence anyone would start to measure the length of the edges, in geometry is called a point. Such a point cannot have any dimensions at all, being only a position relative to the body." This also is illustrated by the angles of a cube.

Thus it is seen that only bodies have a direct existence, that neither surface, line, nor point, exists independently, but that these ideas depend on the existence of the bodies, and are the component parts of the conception of the limits of their dimensions.

Thus we see that when geometry considers the limits of the dimensions of the bodies, the conceptions of superficies, line, and point are necessary consequences of these considerations, and are legitimate subjects for scientific research; at the same time, these conceptions or ideas do not subject the science to the objections already mentioned as being suggested by Euclid's faulty exposition.

The other defect in Euclid's books, the absence of any information as to the relation between the diameter and circumference of a circle, has been the cause of much more error. Euclid being the only light for thousands who have studied geometry, and as his books contained no information, the impression became general that the problem of ascertaining the proportion was insoluble, or at least had not, in Euclid's time, been solved. As the importance of this problem was evident to every one, it is not to be wondered at that many persons, ignorant of the labors of Archimedes, Metius, Van Ceulen, and others, have attempted its solution, to supply this, as they supposed, missing link in geometrical science. Few well informed persons have wasted their time in this direction, but the labor has been bestowed entirely by the ignorant, who, misled by a certain degree of self-conceit, imagined that they have discovered some new properties, which they attempted to use for the solution; the number of such would-be discoverers is very large; and as each went on his own erroneous road, it is not to be wondered at that each reached a different result; and as the premises of each were false, their results were every one inaccurate.

If the method of Archimedes (who first enclosed the circumference of the circle between circumscribed and inscribed polygons of 96 sides, and so found the limits between which the true circumference must be situated) could have been inserted in the books of Euclid, or had been appended to them, the world would have been saved from all the agitation in regard to the quadrature of the circle, and much valuable time would have been saved. But Archimedes lived after Euclid, and so the books of Euclid represent the state of geometrical science before the time of Archimedes; and their continued use in their original condition, for many centuries, has been nothing less than a great misfortune to thousands of students of geometry.

Lacroix, in his "Geometry," published in France in the beginning of this century, first gave a complete logical essay on inscribed and circumscribed polygons, with the method of calculating their peripheries and the peripheries of polygons of double the sides; and by continually doubling, he enclosed the circle in continuously narrowing limits. His method was not new, but he had the merit of so explaining it to beginners that, for its comprehension, a knowledge of only the first books of Euclid was necessary. His method has been adopted by others, and no one who has studied geometry from the books of Lacroix or his imitators can fall into the absurd error that the relation in question is an unknown quantity. We say "absurd error," because new light has been shed upon this subject from various sides, and mathematicians agree as to the figures expressing the relation, which are better known than those of any other irrational quantity; and the calculation has been made to 600 places of decimals, which shows much greater progress than has been made in ascertaining the square root of 2 or the square root of 3, problems which are apparently much simpler than the measurement of the circumference of the circle.

THE FORM AND USE OF CALIPERS.

The use of calipers, in finishing work to a driving fit or a working fit, is a subject of great interest to the general machinist, and a few practical instructions upon the construction and application of calipers will be found useful.

If we notice the standard gauges made by makers of reputation, we shall find them to be, as compared to ordinary calipers, very heavy and strong, the object in thus making them being to prevent them, as far as possible, from springing. We say as far as possible, because deflection always takes place to some extent. Messrs. J. Morton, Poole & Co., of Wilmington, Del., demonstrated this deflection by a very simple experiment. They made a gauge of about 3 inches between the points, its form being that of a crescent, with the points turned towards each other; the width of the gauge at the middle was about $1\frac{3}{8}$ inches, the thickness of the steel being about $\frac{5}{16}$ inch. They made a wire inside gauge to fit the outside gauge so delicately that, if the outside one were held with the two hands, holding the gauge near the points, the inside one would be just sustained by the friction of contact of the outside one; while, if the latter were held in the centre by grasping with the thumb and finger, the inside gauge would fall, thus proving the deflection of the outside gauge by reason of its own weight.

This spring is usually the great disturbing element in taking an exact measurement, and it is here that inaccuracy is induced. To measure correctly with either inside or outside calipers, they must be set so that their contact with the work is scarcely if at all discernible. If we require to set inside and outside calipers to make a working fit, we must bear in mind that, if the outline of the work measured by the outside calipers is of exactly the same diameter as that of the hole into which it is to fit, the one will not enter the other; or, in other words, a pin must be smaller than the hole into which it is to go, in order to have a working fit. The amount to which it must be smaller is a measurable quantity, which is allowed for in solid male and female gauges. In the case of calipers, however, we proceed as follows: First, the points of the outside calipers should have a perfectly even contact when put together, or they may be slightly rounding in their width, as many prefer. Looking at the calipers with the flat sides of the legs towards you, the points should not be rounding, but should be shaped as follows: First, file the points to butt squarely and flat together when closed, and then open the legs and bevel off the end on the convex side to an angle of about 45°, leaving the extreme projecting point face about 1-32 inch wide. Then take a small smooth file, and carefully round over the points, and then harden them to a light purple. The object of making them of this shape is that the part of the points in contact, when measuring different diameters, will always remain the same; whereas such is not the case when the points are rounded, as is often seen in calipers. So, likewise, if the bevel at the points is placed upon the concave side of the points when the calipers are opened wide, the nearest point of contact will be on the bevel instead of at the points, rendering it difficult, in the inside calipers, to find those nearest points. The inside calipers should, instead of having the ends bent around to a curve, have them straight, and standing at an angle of about 45° to the main body of the leg. The part standing at an angle need not be longer than 5-16 inch on a pair of calipers 7 inches long; and the bevel at the points should, in this case, be on the short side of the angle, so that, no matter whether calipers are used upon a small or a large bore, the extreme points will always have contact with the work, and will always stand the furthest away from the centre of the joint. The advantage in this latter point is that we can measure clear to the end of a recess; whereas, if the points are bent around, the curve will, when the calipers are opened at all wide, prevent the points from passing to the back of the recess.

In measuring with the outside calipers we hold them by the joint in the right hand, between the finger and thumb. We then place them upon the work, steadying one leg of the calipers and detaining it in a fixed position by resting it, near the point, against both the work and one finger of the left hand, usually the forefinger. We then move the calipers so that the other leg traverses very slowly over the work, and watch very minutely how near the point approaches to the work. If the latter offers a sensible resistance to the free passage of the caliper point, on round work, we must open them; and when so set that the point will just pass over the work without having perceptible contact, we may try to move that point a little laterally. If we find that the least lateral movement causes contact, while there is one point at which contact is not discernible, the calipers are set. To apply the inside calipers, we hold them in the same manner as above, adopting the same means with the forefinger to hold one point upon the work in a state of rest; while the other point is set so that it is barely perceptible, upon very close examination, that it touches the work. We then hold the inside calipers so that one inside and one outside points contact at the middle of the points, while we pass the other point of the inside calipers past and about the other point of the outside calipers; and when the calipers, so adjusted, will just barely touch each other, the work will be of a working fit, providing it is turned and bored true.

The only difference from this arrangement for a driving fit is that the outside calipers must, instead of being set to just escape the work, be made to have very fine contact with the same. The allowance for a driving fit is so small as to be barely perceptible with a very careful adjustment and manipulation of the calipers, while, for a working fit there must be a perceptible difference, the contact with the inside calipers being more perceptible than that of the outside ones with the work. Here, however, we must remark that the length of the work is an element of consideration, because the standard of truth and parallelism, incidental to such work as is usually measured with calipers, has a great deal to do with this question. For example, we know of no means of boring that will produce so smooth and true a hole as we can finish with a lap; as a consequence we can practically appreciate that there are upon tool-finished work projections, as well as an uneven surface, and in a driving fit these projections act as elements to conform the fit of one part to the other. Suppose, for example, we carefully bore out a hole, 1 inch in diameter and $\frac{1}{2}$ inch deep, the difference in diameters necessary to a driving or a working fit will be almost inappreciable by the closest application of the calipers; and a very slight amount of hand labor, in forcing the one into the other by rubbing them together, will convert a driving into a working fit, the difference being in this case due to a compression of the high spots of the surfaces of the metal. If the surfaces are positively smooth and even, they will form mirrors. If, on the other hand, we take a piece of work, 3 or 4 inches long, the amount of metal on the surfaces which (even with the smoothest of cuts, as ordinarily taken) stands above the bottom of the tool marks, is sufficient to give the parts a driving fit. To appreciate this fact, it is only necessary to carefully turn in a good lathe a piece of iron, say 2 inches in diameter and 4 inches long, and then take a very fine French file and draw file it across the turning marks.

In using calipers upon flat surfaces, it will be found that the inside calipers can be adjusted finer by trusting to the ear than the eye. Suppose, for example, we are measuring between the jaws of a pillow-block. We hold one point of the calipers stationary, as before, and adjust the other point, so that, by moving it very rapidly, we can just detect a scraping sound, evidencing contact between the calipers and the work. If, then, we move the calipers slowly, we shall be unable, with the closest scrutiny, to detect any contact between the two.

In measuring flat work with outside calipers, we must always so adjust them that they barely touch the work; while, at the same time, one point being detained in a state of rest, the other will not move in any direction without positive contact, and this will give a driving fit. For a working fit, the outside calipers may be set so that they are free from contact, and have a barely distinguishable movement. In all cases, however, the truth and smoothness of the work is an important element.

Cast Iron Roofs.

Iron is more used for architectural purposes in America than elsewhere, but not always in such a manner as to render the building fireproof. While corrugated iron roofs are an excellent protection against sparks, they yield too readily to any more intense heat. The Germans, who have generally employed tiles, and make the buildings themselves capable of sustaining such roofs, and even heavier ones, are now introducing cast iron plates for roofs. Those made at the Grœditz Iron Works weigh from 35 to 44 ozs. each, and cover a surface of 8x10, or about 80 square inches, making the weight 4 to 5 lbs. per square foot, or 25 kilogrammes per square meter. A square meter of roofing slate weighs 25 to 30 kilogrammes, and of tile 57 to 60 kilogrammes. The plates have projecting edges so they fit very tightly, and are held in place by 2 wire nails beneath the lap.

Discovery of a New Pink Coral Bed.

The U. S. Steamer Gettysburg, while on her way from Fayal to Gibraltar, recently made a discovery of considerable importance, in the shape of an immense coral bank (hitherto totally unknown), in latitude 36° 30', longitude 11° 28'. Partial surveys were made, and the least depth of water noted was 180 feet, which in mid-ocean is very significant. Twenty miles west of the bank the sounding line marks 16,500 feet, and between the bank and Cape St. Vincent, 12,000 feet. The commander of the Gettysburg believes that in some portions the coral rises to the surface. How such a reef, in a part of the ocean which is constantly traversed by vessels, can have remained undiscovered is almost inexplicable. It is also stated that the bank is rich in valuable coral of light pink shades of color.