HOW THE EARTH IS WEIGHED AND MEASURED.

LECTURE DELIVERED AT THE STEVENS INSTITUTE OF TECHNOLOGY, BY PROFESSOR C. A. YOUNG, OF DARTMOUTH COLLEGE.

It may seem paradoxical to state that the earth is the least accessible to us of all the heavenly bodies; but the fact is that we possess more accurate information concerning the surface of other bodies, such as the moon, for example, than we do of our own planet. There are six cardinal facts connected with the earth. 1. It hangs freely in space. 2. It is approximately spherical, having a diameter of about 7,912 miles; 3. It weighs about six sextillions of tuns, or would if the operation could be performed on its own surface, by bringing up one basketful after another to the surface and weighing it. 4. Its density is about 5·55 times that of water. 5. It rotates on its axis once a day. 6. It revolves about the sun once in a year.

The spherical form of the earth is proved by the shape of the shadow it casts upon the moon in an eclipse of the latter, and by the fact that we see the masts of a ship before the hull comes in sight. There are several other proofs, which it would take too long to describe; but the above are sufficient. The rotation of the earth is proved in various ways. If the earth did not rotate, a falling body would move in a straight line towards the center of the earth. In fact a body, however dropped, say from the top of a tower, will fall a little to the east of the vertical line. The reason is that the top of the tower moves faster than the bottom because it has a larger circle to describe in the same time, and the body dropped partakes of that motion. At a hight of 500 feet, this easterly deviation amounts to about an inch and a quarter. On the same principle a cannon ball will not preserve its true direction, but will be influenced by the rotation of the earth. The direction of the winds is also influenced in an important manner. Suppose a wind to start with a direction due north and south, and to keep that direction constantly. Then the earth's rotation from west to east will cause the wind to come more and more from the eastward, and it will change to northeast, east, southeast, south, southwest, west, and northwest. All this takes place without any real change in the direction of the wind; the change is only apparent, and because our position has changed. In this way the wind veers around in the direction of the hands of the clock about five or six times a year in the northern hemisphere. When there is any change it is due to a disturbance, and a storm is to be looked for. Storms usually rotate in the opposite direction. The lecturer proceeded to show how the rotation of the earth is demonstrated by means of Foucault's pendulum, which consists of a heavy globe of metal suspended by a wire in a frame, which may be rotated by means of a crank. When the pendulum is set in motion, it is found to swing in the same direction, no matter how much the frame is rotated. Now suppose such a pendulum to oscillate for several hours; it will constantly keep in the same plane, but the earth in the meantime carries the table under the pendulum around from west to east, and the pendulum will appear to have changed its plane in the opposite direction. This change is made apparent by means of a graduated circle on the table under the pendulum. In order to make accurate experiments, an extremely long wire and a heavy ball are used; and to insure a perfectly regular motion in one plane only, without any sideward swing, the ball of the pendulum is pulled back by means of a string, which is then fastened and burnt off. To keep it in motion for hours, an electro-magnetic apparatus is employed. At the pole the plane of oscillation would appear to move all around the circle, while at the equator it would continually coincide with the meridian, notwithstanding the rotation of the earth, and would appear unchanged. In latitudes between the equator and the poles, the apparent motion would be proportional to the sine of the latitude

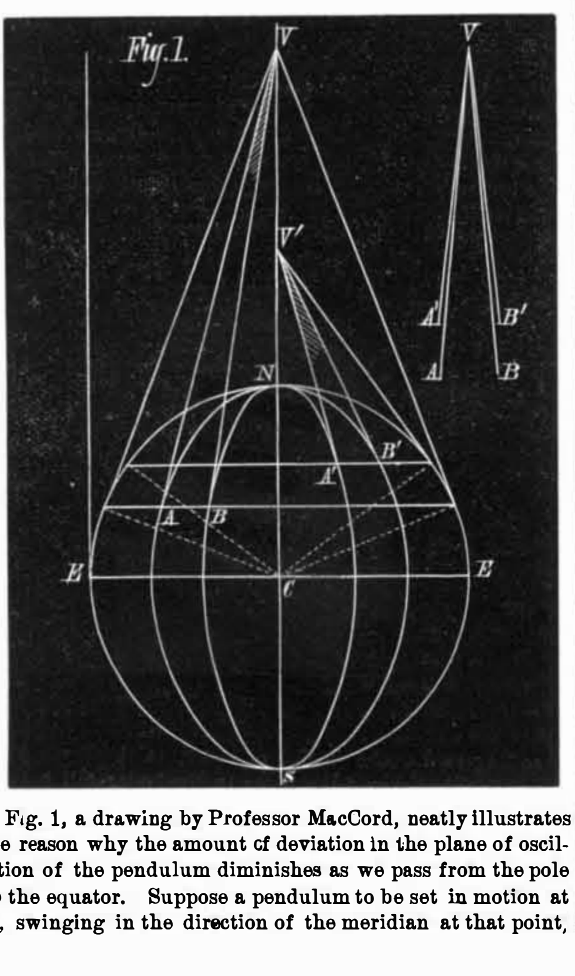
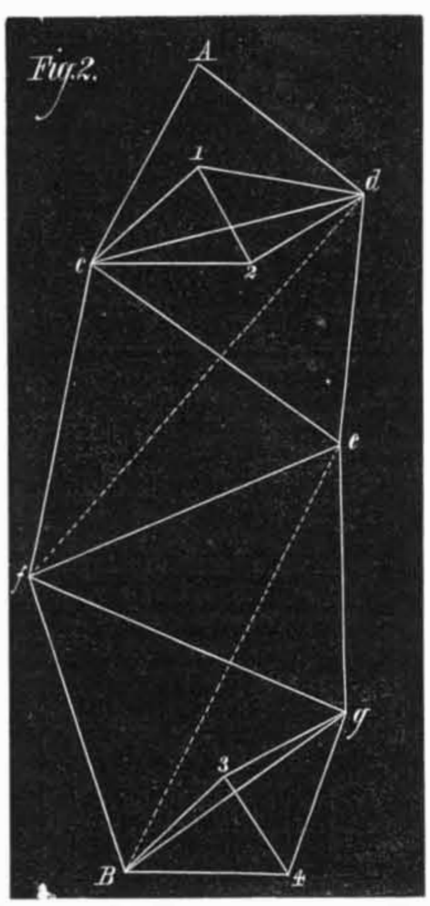


Fig. 1, a drawing by Professor MacCord, neatly illustrates the reason why the amount of deviation in the plane of oscillation of the pendulum diminishes as we pass from the pole to the equator. Suppose a pendulum to be set in motion at A, swinging in the direction of the meridian at that point, which direction is represented by the tangent, A V. It will tend to preserve this direction, when it is carried to B by the rotation of the earth; so that, instead of oscillating in the direction of the meridian at that point, represented by the tangent, B V, it will differ from it by the angle, AVB, included between the two tangents. Now, on taking a parallel of latitude nearer the pole, the angle, A′ V′ B′, formed by the tangents drawn to the two successive positions of a pendulum taken on the same meridians as before, will be greater; or in other words, the displacement of the plane of oscillation of the pendulum will be greater for the same amount of the earth's rotation on the parallel of A′ B′ than on A B. The angles, A V B and A′ B′ V′, which appear only as projections in the main figure, are shown in their true relation to the right. It will be noticed that the tangents drawn to the different meridians, at their intersections with the parallels of latitude, form the elements of a cone, the apex of which recedes further and further from the center as we approach the equator, and that therefore the angles between these elements become less and less. At the equator, all the tangents, as E I, being perpendicular, they would form a cylinder and not a cone; and as they are of course all parallel, the direction of the pendulum would not be changed in passing from one to the other. Another instrument for showing the rotation of the earth is the gyroscope, consisting of a well poised wheel, which continues to gyrate in one direction, but seems to describe a circle with its axis because the earth moves under it. The size of the earth has been pretty accurately determined. The following are the results obtained respectively by Bessel, Airy, and Clarke:

	Bessel.	Airy.	Clarke.
Polar radius in feet.....	20,853,662	20,853,810	20,853,429
Equatorial radius in feet	20,923,596	20,923,713	20,923,161
Ellipticity...	$\frac{1}{299 \cdot 15}$	$\frac{1}{299 \cdot 33}$	$\frac{1}{300 \cdot 65}$

According to Charles, the equator is also elliptical, and his measurements of it are as follows: Semi-major axis 20,926,350 feet (longitude 15° 34′ E.): semi-minor axis 20,919,972 feet (longitude 105° 34′ E.): difference 6,378. Equatorial ellipticity, $\frac{1}{3275}$.



Now in order to determine the dimensions of the earth so accurately, numerous arcs of meridians had to be measured. For this purpose two stations, say two hundred miles apart, are selected, and their latitude is first determined. This is the only part of the whole operation which properly belongs to astronomy. The latitude of a place is its distance from the equator reckoned on the meridian, or, which is equal to the same thing, the altitude or distance of the celestial pole above the horizon. This latter may be measured either by observing the same star at its highest and lowest points, or by observing the pole star, which describes a small circle about the celestial pole, and crosses the meridian twice in each revolution. These observations are made with the transit instrument and the zenith telescope. Having thus determined the true position of the two stations on the earth's surface, we proceed to the measurement of the distance between them. This is done by means of what is called triangulation. A piece of level ground, four or five miles long, is selected to get a base line, *c d*, Fig. 2, which is very accurately measured. Then a third station, *e*, is selected, and the angles it forms with the two ends of the base line are measured with a theodolite. Then, having one side and the angles, the other sides can be easily calculated. In the same way we compute the distance of the stations, *f* and *g*, and finally B. Usually hilltops are selected for the intermediate stations, and the observation of the angles is facilitated by the reflection of a beam of sunlight by means of a mirror, called a heliotrope, into the measurement. The observations are repeated many times to reduce the errors as much as possible; two feet in two hundred miles is about the limit of error allowed.

The measuring rods used in the United States Coast Survey are made of iron and brass in such proportion as to compensate for the elongation and contraction due to temperature. One is placed on tressles, and the other is carried forward and made to touch the first very accurately. The moment of contact is indicated by a spirit level connected with the rod by means of levers.

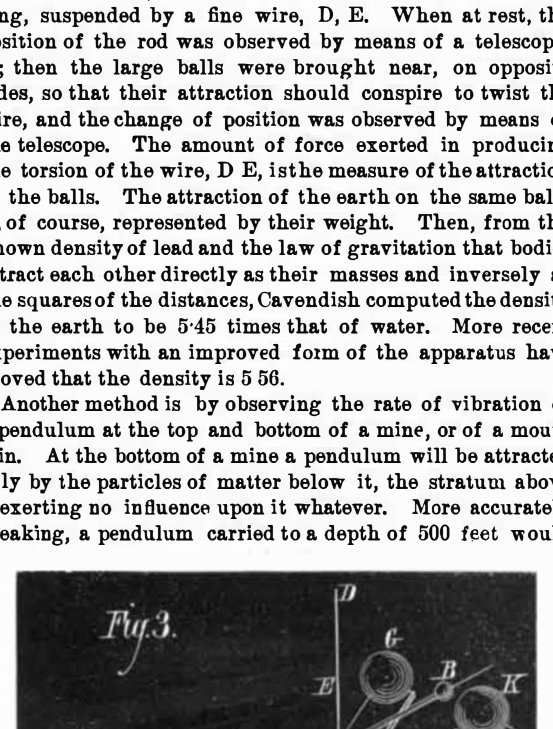
The length of a degree depends of course on the shape of the earth's surface, and the clock furnishes us with a ready means of determining that shape. The vibration of the pendulum is due to the attraction of gravitation, and this becomes greater as we approach the center of the earth. Hence a clock will go faster at the poles than at the equator. It will gain about four minutes, three of which are due to the centrifugal force as has been found by calculation, and one is due to the form of the earth. Therefore, in order to determine the form of the earth, or, in other words, the distance of the surface from the center at different points, all that is necessary is to carry the same pendulum to those points and observe the rate of variation. The best instrument for this purpose is Kater's pendulum, based on the principle (of Huyghens) that the point of suspension and the center of oscillation are interchangeable. It is adjusted by means of sliding weights until it keeps the same time, when it is suspended by its ordinary point of suspension, or turned upside down and suspended by its center of oscillation, which is also provided with a knife edge. This instrument is hung up in front of an astronomical clock beating seconds, and their rate of variation is ascertained by observing them through a telescope and noting the coincidences. It has been found that the degrees are about one seventh longer at the poles than at the equator. Although the accuracy of these measurements is often extolled, it not unfrequently happens that an error of two or three hundred feet is made in determining the latitude of places. Such errors are not the fault of the methods or of the observers, but are due to the variations of the direction of the plumb line caused by the attraction of mountains or dense rocks at the places in question. In the work of the United States Coast Survey, the average error is one hundred and twenty-five feet.

We are enabled to compute the weight of the earth by first determining its density; and this is done by comparing the attraction of the earth upon some object with the attraction of a body of known mass upon the same object.

Dr. Maskelyne accomplished this by determining how much a plumb line was deflected from its normal direction by Mount Schehallian, in Scotland. This deflection was found to be twelve seconds. If the mountain had been as dense as the interior of the earth, the deflection would have been twenty-one seconds. The mean density of the mountain was ascertained, by numerous borings and actual determinations, to be two and three fourths times that of water. Hence the density of the earth is 12 : 21 : : 2·75 = 4·81.

Cavendish, in 1798, compared the attraction of the earth with two lead balls, F and G, Fig. 3, each a foot in diameter. Two small lead balls, A and B, upon which the attraction was exerted, were attached to a wooden rod six feet long, suspended by a fine wire, D, E. When at rest, the position of the rod was observed by means of a telescope, T; then the large balls were brought near, on opposite sides, so that their attraction should conspire to twist the wire, and the change of position was observed by means of the telescope. The amount of force exerted in producing the torsion of the wire, D E, is the measure of the attraction of the balls. The attraction of the earth on the same balls is, of course, represented by their weight. Then, from the known density of lead and the law of gravitation that bodies attract each other directly as their masses and inversely as the squares of the distances, Cavendish computed the density of the earth to be 5·45 times that of water. More recent experiments with an improved form of the apparatus have proved that the density is 5 56.

Another method is by observing the rate of vibration of a pendulum at the top and bottom of a mine, or of a mountain. At the bottom of a mine a pendulum will be attracted only by the particles of matter below it, the stratum above it exerting no influence upon it whatever. More accurately speaking, a pendulum carried to a depth of 500 feet would



vibrate as though it were on the surface of a sphere having a radius 500 feet shorter than that of the earth. Since the density of the stratum above the pendulum can be found by experiment, we are enabled to deduce that of the earth by a simple calculation.

From the dimensions of the earth already given, its volume is found to be about 260,000,000,000 cubic miles, and its weight six sextillions of tons, which, when written out, will present the formidable appearance of 6,000,000,000,000,000,000,000 tuns. C. F. K.