SECOND HONORABLE MENTION ESSAY ON THE FOURTH DIMENSION.

BY "PLATONIDES" (F. C. FERRY.)

(Judges—Prof. Henry P. Manning of Brown University, and Prof. S. A. Mitchell of Columbia University.)

The schoolboy early becomes familiar with linear measure, square measure, and solid or cubic measure. He understands them respectively as "the measurement of lengths," "the measurement of surface which depends on length and breadth taken conjointly," and "the measurement of volume which depends on length, breadth, and height all taken together." The first involves one dimension, length; the second, two mutually

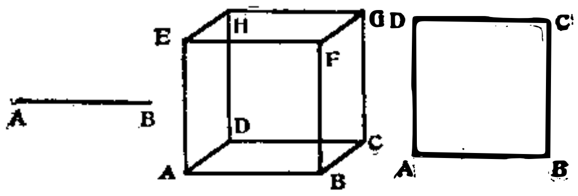

FIG. 1.

perpendicular dimensions, length and breadth, multiplied together; and the third, three dimensions, each perpendicular to the other two—length, breadth, and height, all multiplied together. Let the units of these three kinds of measure (e. g., foot, square foot, and cubic foot) be represented by a line *AB*, a square *ABCD* with that line as side, and a cube *ABCD-G* with that line as edge and that square as base (Fig. 1). The unit *AB* may be regarded as made up of an indefinitely large number *M* of points arranged continuously from *A* to *B;* the square *ABCD* then contains $M\times M=M^2$ points; and the cube *ABCD-G* contains $M\times M\times M=M^3$ points. One can go from any point in *AB* to any or every other point therein by following the one fixed direction of *AB;* similarly, from any point to any or every other in *ABCD* by moving in the two fixed directions of the bounding lines; and likewise in *ABCD-G* by moving in the three fixed directions of the bounding lines (direction forward or backward being regarded as the same in every case). Hence, with regard to motion from one point to another, the first unit is one-dimensional, the second, two-dimensional, and the third, three-dimensional.

Man can make no motion which cannot be resolved into a combination of three mutually perpendicular directions; he can reach no place which cannot be

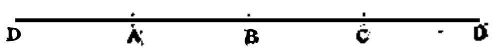

FIG. 2.

reached by going north or south, east or west, and upward or downward; he can find no point in a room which cannot be found by moving in the direction of the length, breadth, and height of the room. Sight reveals two dimensions directly, the breadth and the height of the object beheld, while the third dimension, the distance of the object, is estimated by means of the muscular turning of the eyes to focus them on it. No sense calls for a fourth direction, perpendicular to the other three; in fact, all of man's experience leaves him satisfied with three dimensions.

Leaving experience behind and reasoning wholly from analogy, the fourth dimension is introduced as follows: Four-dimensional measure depends on length, breadth, height, and a fourth dimension, all multiplied together. It involves four linear dimensions, each perpendicular to the other three; consequently the fourth dimension is at right angles to each of the three dimensions of the three-dimensional measure. Its unit must have *AB* as edge, the square *ABCD* as face, and the cube *ABCD-G* as base. It contains $M\times M\times M\times M=M^4$

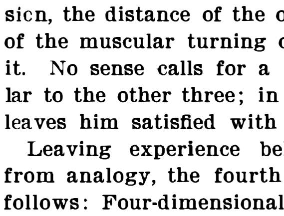

FIG. 3.

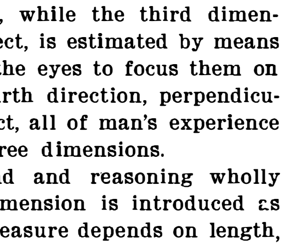

FIG. 4.

points. To travel from any point to any or every other point in it is possible by moving in the four fixed directions of its bounding lines.

The square *ABCD* (Fig. 1) is derived from the line *AB* by letting *AB* with its *M* points move through a distance of one foot in a direction perpendicular to the one dimension of *AB;* every point of *AB* in this motion describes a line, and *ABCD* contains, therefore, *M* lines, as well as M^2 points. The cube *ABCD-G* is derived from the square *ABCD* by letting *ABCD* move one foot in a direction perpendicular to its two dimensions; its *M* lines and M^2 points describe respectively *M* squares and M^2 lines; accordingly *ABCD-G* contains *M* squares, M^2 lines, and M^3 points. Similarly, the four-dimensional unit is derived from the cube *ABCD-G* by letting that cube move one foot in a direction perpendicular to each of its three dimensions, i. e., in the direction of the fourth dimension; its *M* squares, M^2 lines, and M^3 points describe respectively *M* cubes, M^2 squares, and M^3 lines; accordingly the four-dimensional unit contains *M* cubes, M^2 squares, M^3 lines, and M^4 points. Considering the boundaries of the units, *AB* has two bounding points, *ABCD* has four, *ABCD-G* has eight—four each from the initial and the final positions of the moving square—and the four-dimensional unit has 16—eight each from the initial and the final positions of the moving cube. Of bounding lines, *AB* has one (or is itself one), *ABCD* has four, *ABCD-G* has twelve—four each from the initial and the final positions of the moving square, and four described by the four bounding points of that square; and the four-dimensional unit has 32—twelve each from the initial and the final positions of the moving cube, and eight described by the eight bounding points of that cube. Similarly, of bounding squares, *ABCD* has one (or is itself one), *ABCD-G* has six—one each from the initial and the final positions of *ABCD*, and four described by the bounding lines of the moving square—and the four-dimensional unit has 24—six each from the initial and the final positions of the moving cube and twelve described by the bounding lines of the moving cube. Finally, of bounding cubes, *ABCD-G* has one (or is itself one), and the four-dimensional unit has eight—one each from the initial and the final positions of the moving cube, and six described by the bounding squares of the moving cube.

If the bounding lines of the square *ABCD* are supposed to be made of a continuous wire and that wire is cut at *D*, the boundary may then be folded down into line with *AB*, forming a one-dimensional figure

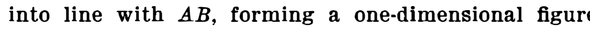

FIG. 5.

(Fig. 2) of four linear units. The original linear unit *AB* has one linear unit at either side of it and an extra one, *CD*, beyond on one side. If the cube *ABCD-G* has its bounding squares supposedly made of a continuous sheet of tin and that sheet is cut along the lines *EF*, *GH*, *HE*, *AE*, *BF*, *CG*, and *DH*, the square faces can be folded down to form a two-dimensional figure of six squares. The square *ABCD* has a square at each side of it and an extra one, *EFGH*, beyond on one side (Fig. 3). Likewise, if the four-dimensional unit has its bounding cubes made of connected solid wood and this wood is cut through the appropriate planes, the cubes can be folded down to form, by analogy, a three-dimensional figure of eight cubes. The cube *ABCD-G* has a cube at each side of it and an extra one beyond on one side (Fig. 4). These eight cubes, now forming a three-dimensional figure, constituted the boundary of the four-dimensional unit.

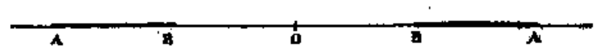

FIG. 6.

The following table shows the results obtained for the contents and the boundaries of the four units considered:

CONTENTS.

	Points.	Lines.	Squares.	Cubes.
One-dimensional unit	M	1	0	0
Two-dimensional unit	M^2	M	1	0
Three-dimensional unit	M^3	M^2	M	1
Four-dimensional unit	M^4	M^3	M^2	M

BOUNDARIES.

	Points.	Lines.	Squares.	Cubes.
One-dimensional unit	2	1	0	0
Two-dimensional unit	4	4	1	0
Three-dimensional unit	8	12	6	1
Four-dimensional unit	16	32	24	8

The reasoning used is capable of extension at once to units of five, or even more, dimensions.

If the one-dimensional unit is extended indefinitely to the right beyond *B* and to the left beyond *A* so that its length becomes greater than any number one can name, it represents a one-dimensional space. Similarly, the indefinitely great extension, equally in every dimension, of the other units gives a representation respectively of two-dimensional, three-dimensional, and four-dimensional spaces.

The one-dimensional unit is separated from the rest of the one-dimensional space in which it lies by two points, the two-dimensional unit from the rest of its two-dimensional space by four lines, the three-dimensional unit from the rest of its space by six squares, and, similarly, the four-dimensional unit is separated from the rest of the four-dimensional space in which it lies by eight cubes. To inclose an object of any number of dimensions in space of the same number of dimensions demands, in one-dimensional space, two points, in two-dimensional space, at least three lines, in three-dimensional space, at least four planes, and, in four-dimensional space, at least five three-dimensional spaces.

As with the units, so with the spaces, any point can be reached from any other in the same space by moving in as many fixed directions, each perpendicular to the rest, as that space has dimensions.

Time represents a one-dimensional space; for it proceeds in one direction only from an indefinitely remote past to an indefinitely distant future (Fig. 5). The present is a point traveling through time (or

FIG. 7.

allowing time to slip past it) with uniform velocity; and any point in time can be reached by traveling through a definite distance (in years, months, etc.) from one chosen fixed point (e. g., the birth of Christ).

Any portion of the earth's surface, regarded as a plane, represents a portion of a two-dimensional space; and the two fixed directions are those of latitude and longitude. An illustration of three-dimensional space is that space—to man's perceptions—in which the universe is placed. Man can find no illustration of a four-dimensional space.

If two lines, *AB* and *B'A'*, in the same one-dimensional space are symmetrical about a point *O* of that space (Fig. 6), *AB* cannot be so moved in that space that the corresponding points shall coincide (*A* with *A'*, *B* with *B'*, etc.). To effect such coincidence, it is necessary to rotate *AB* through two-dimensional space about *O* as a center; or, roughly speaking, *AB* must be taken up into two-dimensional space, turned over, and put down on *B'A'*. Likewise, if two triangles, in the same two-dimensional space, are symmetrical with respect to a line (Fig. 7), such coincidence of corresponding points and lines can be effected only by rotating one triangle through three-dimensional space about the line of symmetry; or, roughly speaking, one triangle must be taken up into three-dimensional space, turned over, and put down on the other. Again, if two polyhedral figures in the same three-dimensional space are symmetrical with respect to a plane (Fig. 8), coincidence of corresponding points, lines, and planes can be effected only by rotating one polyhedral figure through four-dimensional space about that plane; or, roughly speaking, one of the polyhedral figures must be taken up into four-dimensional space, turned over, and put down on the other. A right hand and its reflection (a left hand) in a mirror are symmetrical with respect to the plane of the mirror and rotation about that plane would effect coincidence. Such rotation would make a right glove become a left glove; or, roughly speaking, a right glove tossed up in the direction of the fourth dimension and turning over there will fall back a left glove.

The inability of man to locate the fourth dimension or to detect the existence of a four-dimensional space, even if it be close at hand, is comparable with the inability of a two-dimensional man, inhabiting a two-dimensional space, to locate the third dimension or to detect the existence of three-dimensional space, even though his own space might be part of it, as a plane is part of a solid. Suppose the two-dimensional

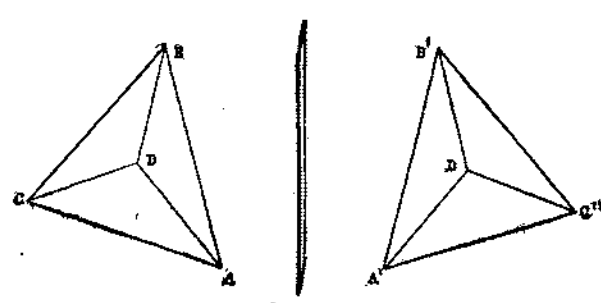

FIG. 8.

space represented by this page to be inhabited by two-dimensional beings. They have length and breadth, can move in those two dimensions, and are supposedly conscious of them. They have no thickness, cannot rise from the paper or sink beneath it and are unconscious of any dimension in such a direction; they have no "upward" and no "downward." Let them have intelligence concerning all within their space to the extent that man is intelligent regarding his universe; let them possess houses and barns, and in general let their life be as rich as may be. Their houses and barns will have no roofs and no floors, for the bounds of the space itself alone are there. Three lines are sufficient to inclose any object in their world, and the flat-man himself is exposed only along his polygonal contour; the interior of his polygon—his own interior—is to be reached only through this contour, for there is no "above" and no "below" within his cognizance. To convince him that a third dimension of "upward" and "downward" exists, touching and leading from even the interior of his polygon—his own internal parts—would be a hopeless task. Even if he accepts the arguments from analogy as to the properties of such a dimension, he would rebel at the idea of looking within himself to find it. Yet, even there, at right angles to the two dimensions which he knows, it is to be found—as well as everywhere else in his space. And, similarly, within himself, quite as much as anywhere else, must man look if he is to find the fourth dimension.

Were one to explain to this flat-man that a three-dimensional being, approaching from the direction of that unknown third dimension, could reach within his most securely locked barn and remove its contents without opening a door or breaking a wall—or could touch the very heart of the flat-man himself without piercing his skin—the flat-man might still be none the nearer to an appreciation of the third dimension. Equally impossible is it for man to understand from what direction a four-dimensional robber must come to steal the treasures from the soundest vault without opening or breaking it—or by what way of approach the four-dimensional physician would reach to touch the inmost spot of the human heart without piercing the skin of the body or the wall of the heart; yet the route of such a robber and of such a physician lies along the fourth dimension. By that route must come the four-dimensional being who is to remove the contents of the egg without puncturing the shell or drink the liquor from the bottle without drawing the cork. Such four-dimensional creatures, inhabiting a space containing the three-dimensional space where man lives, would constitute the most perfect of ghosts for man's world; and the absence of such ghosts argues against the existence of a four-dimensional space so situated and so inhabited.

Algebra demands that geometry picture all its problems; and since an algebraic problem may contain four or five or more unknown quantities quite as well as any lesser number, algebra demands a four-dimensional, five-dimensional, or higher space for its use quite as imperatively as the spaces of lower dimensions. Perhaps certain phenomena of molecular physics or the mechanical principles of the electric current may find a complete explanation only with the use of the fourth dimension. Perhaps the fourth dimension escapes man's discovery only because the measurements in its direction are always very minute in comparison with the measurements in the three other dimensions. Thus far, however, the space of four dimensions—and all spaces of more dimensions—may be only "the fictitious geometric representation of an algebraic identity."

A NEW SCOTCH ELEVATING FERRY.

BY WILLIAM CARLIN.

The front-page illustration shows the construction of the new elevating vehicular ferry steamer "Finnieston No. 1," recently constructed at Port Glasgow for the Trustees of the Clyde Navigation under the direction of G. H. Baxter, Esq., mechanical engineer. The vessel was launched with machinery aboard and steam up, the illustration being a photograph taken immediately after the launching. The leading dimensions are, length 104 feet, beam 45 feet, and molded depth 12½ feet.

It may be stated that the elevating platform which carries the vehicles has a range of 17 feet and is carried on eight double-threaded buttress screws of forged steel. The screws are hung on collar bearings in cast-steel brackets, which are supported by the framing legs. The platform is built up of H girders connected with massive built steel girders on either side of vessel. The supporting screws are fitted with worm wheels at their lower ends, and mesh with forged steel worms.

A triple-expansion, three-crank engine raises or lowers the main platform. An automatic gear is fitted to this engine, so that the platform may not be raised or lowered beyond its intended travel. A brass gage in the engine room also indicates the position of the platform in feet and inches.

The lower or main deck is of steel plating, and has no projections above 10 inches. As a result, the platform may drop to its lowest level.

Vertical three-crank, triple-expansion engines are used for propulsion, each engine driving two propellers, one forward and one aft, with two thrust blocks fitted on each line of shafting. The engines are controlled from the house on the top of the framing by balanced rods, which actuate the steam valves on the direct-acting steam and hydraulic reversing engines. There are no rudders, the vessel being maneuvered entirely by the propelling machinery. Two reversing handles are situated in the wheel house, one on each side of the man at the wheel.

Correspondence.

THE EGYPTIAN STEAM CULTIVATOR.

To the Editor of the SCIENTIFIC AMERICAN:

In March last I noted a short article in your paper speaking of a steam cultivator by Boghos Pacha Nubar, a farmer in upper Egypt. The description and purpose of this cultivator are both identical with a machine that I was interested in some ten years or more ago, together with a number of others here, a company to promote which was formed, but failed to produce a thoroughly commercial machine. By that I mean that although several machines were built, with from two to six rotary disks or tools, as we call them, owing to lack of capital we were not able to put out a machine that would work day in and day out without a break of some kind. This we know was only a question of capital and experiments.

I write you so as to correct any impression that may exist as to the originality of the machine you speak of; for I may add this principle was patented here in Canada more than twenty years ago, and should you desire to know more of it, the information could be secured. H. J. ROSS.

Montreal, Canada.

THE PROPOSED MONTREAL DRYDOCK.

To the Editor of the SCIENTIFIC AMERICAN:

I read in the press of a proposal to have a floating drydock in Montreal, and the following suggestion flashed into my mind as being most suitable for local reasons: Instead of a floating dock, why not excavate a dock, near the exit of the Lachine Canal, in two sections, one the real drydock with the floor just above the level of high water at Montreal, the other in continuation with floor at level of present harbor bottom level.

Vessels could then run into the lower section, gates be closed, and water run in from the canal through sluices, which would raise the level some 50 feet, or as required; pass the boat into the inner section, close sluices and open outflow, letting the water run out, thus saving pumping, time, and expense.

At the beginning of navigation, perhaps some pumping might be required during the freshets.

This, in my opinion, would be the easiest, cheapest, and most permanent way to build a drydock at Montreal. VICTOR F. CONNOR, L.R.C.P.I.

Great Village, N. S.

THE NUMBER OF OUR ANCESTORS.

To the Editor of the SCIENTIFIC AMERICAN:

May I suggest that Mr. McCullough has not solved the problem put by Mr. Venning?

No one will deny that, shall we say, John Brown had a father and a mother, that his parents each had a father and a mother, and so on, for (all?) previous generations. It would appear, then, to follow that x generations back, John Brown had 2^x ancestors! This is what Mr. Venning says *appears* to be the fact. He does not say it *is* the fact; indeed, he asks for an explanation of the difficulty he is placed in, in not being able to reconcile this apparent fact with common sense.

Would any of your readers explain where the error creeps in? John Brown we know had 2 parents; we know he had 2^2 grandparents; we know he had 2^3 great-parents. Why then is it not true that for the xth generation back he had 2^x ancestors?

Bristol, England. F. C. CONSTABLE, M.A.

SIGNALING TO MARS.

To the Editor of the SCIENTIFIC AMERICAN:

The possibility of signaling to Mars is merely a question of elementary mathematics. That it should have excited such widespread interest and discussion can have astonished no one more than the writer. That in spite of this widespread and in many cases correct exposition by the newspapers, there should still remain some who fail to grasp the elementary principles of the problem is my only excuse for what may seem to many of your readers a wasteful use of valuable space in your paper.

Reference is here made to two communications in your issue of June 26th. The first is by a gentleman connected with Adelphi College, Brooklyn. This criticism is that if the signals were sent when Mars was in opposition they could not be seen. The obvious answer would seem to be: "Then why send them when Mars is in opposition?" There are two positions of the earth in its orbit when it is invisible from Mars. One is when it is between Mars and the sun, and the other when it is on the opposite side of the sun. So it would hardly seem desirable to send signals to Mars under either of these circumstances. Signals could be sent in any other relative position of the two planets, as has been fully explained in several of the daily papers. This statement will also answer the criticism of the gentleman regarding the perfectly logical and correct suggestion of Prof. Wood, that a signal might also be sent by means of a dark area upon a white field.

The first mistake of the other writer is due to the fact that in the cases which he cites, and with which he is familiar, the angular size of the mirror is greater than the angular size of the sun. The object of the silvering of the mirror is to change the direction of the rays of light, but it really has nothing to do with the question in hand, and we may consider for convenience that the mirror has been removed during our experiments, and that the sunlight is simply shining through the hole in the frame. If now we are very near the mirror, so that we see the whole of the sun's disk through the aperture, a 2-inch mirror will transmit just as much light as one ten feet in diameter. If, however, we go to a distance of several miles, the large hole will clearly let through much more light than the small one.

The second mistake of this correspondent is to suppose that the signal would only be seen over a small portion of the surface of Mars. In point of fact, when Mars was distant one hundred millions of miles from the earth, the signal would be seen simultaneously over an area one million miles in diameter. No great accuracy would therefore be required in pointing the mirror, as he seems to suppose.

In closing, the writer would repeat that the proposition of signaling to Mars is merely a question of the most elementary mathematics. It is a problem which any astronomer can work out in ten minutes' time and which involves no uncertainty whatever. When Mars is at a distance of one hundred millions of miles from the earth, a beam of sunlight half a mile square would appear to its inhabitants of the same brightness as a fifth magnitude star. On account of the brightness of the earth, however, it would be quite invisible to eyes resembling our own, unless aided by a powerful telescope. Whether there are the equivalents of human eyes and telescopes upon Mars is another question entirely and has nothing whatever to do with the case.

WILLIAM H. PICKERING.

Harvard College Observatory.

Official Meteorological Summary, New York, N. Y., June, 1909.

Atmospheric pressure: Highest, 30.23; lowest, 29.69; mean, 29.97. Temperature: Highest, 92; date, 25th; lowest, 53; date, 19th; mean of warmest day, 82; date, 24th; coolest day, 56; date, 9th; mean of maximum for the month, 78; mean of minimum, 63; absolute mean, 70.5; normal, 69.1; excess compared with mean of 39 years, 1.4. Warmest mean temperature of June, 72 in 1888, 1892, 1899, 1906; coolest mean, 64 in 1881, 1903. Absolute maximum and minimum of June for 39 years, 97 and 45. Average daily excess since January 1st, 2.0. Precipitation: 3.17; greatest in 24 hours, 0.70; date, 4th and 5th; average of June for 39 years, 3.21. Accumulated deficiency since January 1st, 0.19. Greatest precipitation, 7.70, in 1877; least, 0.86, in 1894. Wind: Prevailing direction, southwest; total movement, 7.014 miles; average hourly velocity, 9.7; maximum velocity, 43 miles per hour. Weather: Clear days, 7; partly cloudy, 12; cloudy, 11; on which 0.01 inch or more of precipitation occurred, 12. Thunderstorms, 22nd, 25th, 27th, 28th.

The Current Supplement.

The opening article of the current SUPPLEMENT, No. 1750, discusses wild animals in captivity. A novel electric locomotive is described and illustrated. The astronomical clock at Lyons is an interesting article by Charles A. Brassler. The indestructibility of matter is treated by Prof. G. Zenghelis. Col. Sir Frederic L. Nathan's admirable paper on guncotton and its manufacture is concluded. W. C. Horsnaill writes illuminatingly on the subject of tidal power. The article should prove most helpful to inventors of tidal and wave motors. A *résumé* of F. W. Lanchester's discourse on aerial flight presented before the Royal Society of Arts is published. An apparatus is described for studying the friction of metals experimentally. An interesting article is that by C. Ainsworth Mitchell on the making of handwriting. Prof. John Joly, the distinguished geologist, contributes an article on radio-activity of deposits and the instability of the earth's crust.

The smelter production of lead in the United States in 1908, as given by C. E. Siebenthal, of the United States Geological Survey, under date of May 24th, was 408,523 tons of 2,000 pounds, against 442,015 tons in 1907, and 418,699 tons in 1906. The production of refined primary lead, which embraced all desilvered lead produced in the country, and the pig lead recovered from Mississippi Valley lead ores, was 396,433 tons, against 414,189 tons in 1907, and 404,669 tons in 1906. The antimonial lead produced was 13,629 tons, and the recovered or secondary lead 18,283 tons. In 1908 the lead smelted from domestic ores was 310,762 tons, and from foreign ores and foreign base bullion (almost wholly Mexican), 97,761 tons.