Correspondence.

A PROBLEM IN ARRANGING NUMBERS.

To the Editor of the SCIENTIFIC AMERICAN:

For quite a long time your correspondence column contained a lot of matter on magic squares. Recently a problem came to my notice which is somewhat different from magic squares, but which must surely be solved by a system. The problem is to arrange the numbers from 1 to 15 in seven different combinations of five rows and three numbers in each row, so that no two numbers will be in the same row more than once. Of course, it is quite apparent that any number can be combined with seven different pairs, but by what system can this or any similar problem be solved? HERMAN S. RIEDERER, Ph.D.

New York city.

CONTROLLING STEAMSHIP ENGINES FROM THE BRIDGE.

To the Editor of the SCIENTIFIC AMERICAN:

I noticed in a recent issue the suggestion by some reader of letting the engineers on ocean-going vessels, and in fact all large steamers, see where they are going, and judge the speed for themselves, as when making a landing.

In this connection I would say that besides the troubles mentioned by you, there would be the even more serious one of having two men, each thinking separately, and each figuring on what to do without knowing what the other will do. For example, in case of an apparently impending collision, the pilot sees that by making a quick turn he can avoid it. The engineer sees the situation, and not figuring on the possibility of steering away, backs his engine. The result would be the vessel would not answer her helm quick enough, with disastrous results.

Now, there is a system of control on a large number of motor boats, in fact on motor boats up to and including 75-footers, called "one-man control," which is nothing more or less than bringing the engine controls to the steering wheel, as on an automobile.

Steering, a very delicate operation, is done by steam. Why, with a similar apparatus, cannot a large vessel be made a "one-man control" boat? There is no great mechanical difficulty to it. And what is more, a steam engine could be entirely controlled by one lever, whereas a gasoline motor takes three—reverse gear, spark, and throttle.

I would like to hear what others have to say on this subject. H. SUSSMAN.

New York city.

Some Curious Number Puzzles.

BY J. F. SPRINGER.

There are many curious things about numbers. Some of these afford the basis for puzzling games. To the uninitiated the results have a more or less wonderful appearance. In the following article it is proposed to instance and explain certain of these puzzles. The first is somewhat similar to a puzzle published by Mr. W. W. R. Ball. The others are new and appear now for the first time.

To begin with—there is the puzzle of the reversed digits. A person is requested to select a number having in it an odd number of digits. He is next asked to reverse the order of the figures, thus producing a second number. Now he is directed to subtract the one number from the other and multiply the result by any number he pleases, cutting off any naughts at the end. Upon obtaining this he is to cross out the final two figures and tell you the resulting number. He will probably be surprised to have you tell him the figures which he crossed out. You do this by annexing two naughts at the end of the number he tells, divide by 99 and note the remainder. This remainder subtracted from 99 yields the number crossed out by him.

Thus suppose the number chosen to be 58463 (an odd number of digits). Reversing this, we have 36485. Subtracting one from the other, we obtain, 21978. Multiplying this by 23 (any number will do), we get 505494. Crossing out the final two digits (94) we are supposed to be told the result—5054. To this we annex two naughts—505400. Dividing by 99 we get for remainder 5. Subtracting this from 99, we get the crossed-out number 94.

To understand the underlying reasons is not difficult. Thus, suppose the original number possessed, say, three figures. It can be represented algebraically by the expression $100x + 10y + z$, when x, y, and z represent the several digits. The reversal of this is $100z + 10y + x$. Subtracting one from the other, we get $99x - 99z = 99\ (x - z)$. This is evidently divisible by 99. This divisibility by 99 will still be true if we multiply by any number we please. The number then from which the two figures are to be crossed out after cutting off naughts at the end, is divisible by 99. Cutting off final naughts will not affect divisibility by 99, In crossing out the final two figures, we are not able to take away more than 99, as this is the largest number containing just two digits. Replacing then the two crossed-out figures by naughts, we have a number which lacks something of being divisible by 99 or does not. In either case, what needs to be added to bring the remainder up to 99 is the number crossed out. If the remainder is naught, then the number crossed out was 99 itself.

Another puzzle where two figures are determined by the performer is the following: A person is asked to select two prime numbers. He is then to add them and square the result. For a moment this is to be set aside. Next, the smaller of the primes is to be taken from the other, and the resulting difference squared. This last result is then to be subtracted from the result laid aside a moment ago. You now request to be told the result. This you divide by 4. You will then obtain a number which can be factored in but one way, yielding two prime numbers. These are the numbers selected at the outset.

To illustrate, suppose the prime numbers selected are 7 and 13. Adding and squaring you get 400. Subtracting and squaring there results 36. Subtracting this last number from 400, you have 364. Dividing this by 4, you obtain 91. This can be factored in but one way—7×13.

To explain the matter algebraically, let x and y represent the two prime numbers. Adding and squaring, we get $(x + y)^2$. Subtracting and squaring, we have $(x - y)^2$. If now we write these expressions out in full and subtract the second from the first, thus

$$(x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)$$

we shall get $4xy$. This is the number told to the performer, corresponding to 396 above. Dividing by 4, we of course get just xy. As both x and y are primes, their product is factorable in just one way—into x and y.

A third puzzle is when a person is requested to select a number. He is then to square it and lay aside for a moment. You then request that he take a number which is one less than the one selected and square this also. He is now to subtract one square number from the other and tell you the result. You then add one to the number he tells you and divide by 2. This will give you the original number.

Again, suppose that instead of subtracting one and then squaring, your friend had been asked to subtract 2 and square. You will then request the two squares to be subtracted the one from the other and the result communicated to you—just as before. You now add 4 and divide by 4, getting for result the number chosen.

Thus in the first case, if the number chosen were 13, the square would be 169. The square of one less than the number $(13 - 1 = 12)$ would be 144. Subtracting, 25 is obtained. This is the number communicated to the performer. Adding 1 and dividing by 2, we get 13—the original number.

In the second case, after squaring 13 and getting 169, we square $13 - 2 = 11$ and get 121. Subtracting this from 169 we obtain 48. Adding 4 and dividing by 4, as directed, we obtain 13.

Algebraically considered, we have for the first case $x^2 = (x - 1)^2$ as the number told to the performer. This is really $2x - 1$. If we call this number N, we have $N = 2x - 1$. Now N is known and we wish to get x. We have therefore $x = \frac{N + 1}{2}$

In the second case, we have $x^2 - (x - 2)^2$, which yields $4x - 4$. Putting $N = 4x - 4$, we get

$$x = \frac{N + 4}{4}.$$

Or, to state the puzzle in a more general form, we may request that the number to be subtracted be A, instead of 1 or 2, as before. We have then $x - (x - A)^2$. This gives $2Ax - A^2$. Putting $N = 2Ax - A^2$ and solving for x we get $x = \frac{N + A^2}{2A}$. By using a formula such as this we may keep up the mystification by continually changing the number to be subtracted. We have simply to remember that we add to the number told us the square of the number subtracted from the original number and then divide by twice this number (but without squaring). Thus if we tell the person to subtract 7 and then to square the result, at the end when he tells us the result of all his operations, we have simply to add 49 $(= 7^2)$ and divide by $2 \times 7 = 14$.

Again, you request some one to square two consecutive numbers and add the results. You then ask that he double this amount and subtract one. You are then to be told the result. If he has performed his operations correctly, this will be a perfect square. You take the square root. This will be the sum of the original consecutive numbers. By subtracting 1, and dividing by 2, you will determine the smaller. The other one is of course obtained by adding 1.

Thus, suppose your friend selects the consecutive numbers 11 and 12. Squaring and adding, he gets $121 + 144 = 265$. Doubling this and subtracting 1, he obtains 529. This is a perfect square, it is the number he tells you. Taking the square-root, you find it to be 23. Subtracting 1, and dividing by 2, you get 11. The original numbers are therefore 11 and 12.

To explain the matter algebraically, we write $x^2 + (x + 1)^2$ as the sum of the squares of the consecutive numbers. The next operation is expressed thus: $2\ [x^2 + (x + 1)^2] - 1$. This gives $4x^2 + 4x + 1$. This is the number told the performer and should be a perfect square. It is. In fact, it is equal to $(2x + 1)^2$. Subtracting 1 from the square root the result is $2x$. Dividing by 2, the smaller of the two consecutive numbers is obtained.

Still another puzzle consists in the following procedure: You ask the "patient" to select a number of any number of digits. He is then to form with the same digits a second number by rearranging the digits in any way agreeable to himself. These two numbers are then to be subtracted, the one from the other, and the result multiplied by 33. You now request that any naughts that may be at the right-hand end be cut off. He is then to cross out the final pair of digits and tell you the number remaining after this is done. You annex two naughts, divide by 99 and subtract the remainder from 99. The result will be the figures crossed out.

This puzzle depends for its success upon the fact that when two numbers formed of precisely the same digits, but differently arranged, yield, upon the one being subtracted from the other, a new number divisible by 9. Upon multiplying this by 33, it will still be so divisible, and will in addition contain 11 as a factor. In consequence of containing 9 and 11 as factors, it will contain 99 $(= 9 \times 11)$ as a factor. The explanation of the remaining procedure is the same as that given in the case of a preceding puzzle. That two numbers having the same digits, but differently arranged, will yield a number divisible by 9 may be seen by considering the following equations:

$$10000w + 1000x + 10y + z - (10000y + 1000x + 100z + w) = 9999w - 9990y - 99z = 9\ (1111w - 1110y - 11z).$$

To illustrate this puzzle, suppose the number selected is 46309. Rearranging the digits we get, say 60349. Subtracting 46309 from 60349, we get 14040. We now multiply by 33 and get 463320. Observing that this has a naught at the right-hand end we cut it off, obtaining 46332. We now cross out the 32 and get 463 as the number which would be told to the performer. He annexes two naughts and gets 46300. This he divides by 99, obtaining 67 for the remainder. He subtracts this from 99 and gets 32—the number crossed out.

A New Fare Box for Pay-as-You-Enter Cars.

A new fare box for pay-as-you-enter cars has made its appearance in New York. It differs in almost every way from the usual contrivances of its kind. In the first place it not only collects and registers nickels, but gives the conductor access to the cash, after it has been registered, and thus renders it unnecessary for him to fill his pockets with change before he starts out on his trip. In the second place, the new fare box enables the conductor to detect mutilated and counterfeit money; for the coin slides down a glass-covered chute and is therefore visible. If the conductor discovers that it is a counterfeit, the coin is mechanically dropped into a special receptacle and returned to the passenger. If the coin is good, it is registered and caused to drop into a cash drawer. Although it is not likely that a dishonest conductor would attempt to abstract so small a sum as a nickel by turning the box upside down, so that the nickel would run out, an automatic gravity closure has been provided which effectually blocks the chute if the attempt should be made, so that the fare must pass through the apparatus in the regular way. The width of the chute is just large enough to accommodate a good nickel. A larger coin cannot enter, and a smaller coin is mechanically returned by the machine. Each conductor is held responsible for the fares registered. Although he has access to the cash collected, the amount turned in must be the amount recorded. Thus a very simple and effective check is provided, without the necessity of giving the conductor a large amount of small change with which to start his trip. The boxes are made of a special composition of metals and weigh only 10 pounds.

Peat fuel will be used in an electric station being erected in Germany. The new plant is being installed in the region to the southwest of Oldenburg, and it lies in the vicinity of extensive peat fields from which the supply is to be secured. The electric plant is laid out on a large scale, and when completed it will be one of the largest in the country. It is to furnish current over a network of power lines which will cover the entire Duchy of Oldenburg, with a radius of 40 miles. Probably the new station will be completed about the end of next year, and in the meantime measures are being taken to find out about what amount of current will be taken in the various cities and communes within the area.