THE FOURTH DIMENSION SIMPLY EXPLAINED

THE FIRST HONORABLE MENTION ESSAY IN THE FOURTH DIMENSION COMPETITION.

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[It was one of the conditions of the Fourth Dimension Competition that the editor of the SCIENTIFIC AMERICAN reserved the right to publish those essays which were considered worthy of honorable mention. In accordance with that condition, the essay which in the opinion of the judges, Profs. S. A. Mitchell and H. P. Manning, was deemed worthy of first place in the honorable mention class is here printed. Prof. Manning does not dissent for the most part from the conclusions of the author of the essay, but believes that the author has not quite fairly treated the arguments of the "fourth dimensionists." Prof. Manning will later offer some detailed criticisms on this point.—ED.]

The fourth dimension has no real existence in the sense in which the external world that we know by means of our senses has real existence. It is a philosophical and metaphysical conception, whose actual existence cannot be demonstrated by observation or by logical reasoning. The existence of the fourth dimension is regarded by some as in a high degree probable, and as furnishing a basis for metaphysical investigation, and a means of explaining some physical phenomena, the occurrence of which, however, is not universally admitted. It may also, like any supposition, true or false, be made the hypothesis for mathematical speculations, which are comprehensible, however, by the very small and select number only who are endowed by nature with the ability to cope with original investigation in the domain of the higher mathematics.

The word "dimension" is more readily explained than defined. All more or less clearly conceive of space as extending indefinitely or infinitely in every direction; and of extension in space there are three "dimensions"—length, breadth, and thickness. Or, in another point of view, having three fixed points from which to reckon measurement, by three dimensions or measurements we can fix exactly the position of any point in space. Thus, if the three fixed points be the center of the earth, one of the poles, and some other point on the surface, as the location of the Royal Astronomical Observatory at Greenwich, the length of the line drawn from the center of the earth to the point in question in space, as a star, however remote, and the latitude and longitude of the point in which the line from the center intersects the surface, will be three dimensions, which fix exactly the position of the point in space, or of the star. Or again, starting from any point in space, we may reach any other point by proceeding successively in three directions at right angles with one another. Thus, moving from the starting point, first the proper distance east or west, then from the point arrived at the proper distance north or south, and finally the proper distance up or down, we reach the second point in question.

In all the ways in which the meaning of the word is thus illustrated we see that we can have no fewer and no more than three dimensions; but the believers in a fourth dimension infer its existence from analogy in one of the following deductive processes:

(1) Conceive, we are bidden, of a space of but one dimension. A being in such a space would be limited to a straight line, which he would conceive as extending infinitely in both directions. His only possible movement would be along this line, and if he encountered another being, neither could pass the other. If he is really within a space like ours, although his perception is confined to one direction only, and a being in our space should lift one of the two beings, and place him on the other side of the first, the latter would lose sight of the other as soon as the lifting took place, and the movement by which the change of position had been effected would be utterly unintelligible to him.

Conceive of a space of but two dimensions, like the flat surface of a table. Beings in such a space could move around one another, but one of them completely surrounded by others would be imprisoned by them. If, as before, the two-dimension space is within our space, and really depends on the limitation of the perceptive faculties of the beings in question, the imprisoned being could be lifted by a being in our space, and set down outside of the beings surrounding him. The latter would lose sight of him during this movement, and could not understand how it had been effected.

From these suppositions of one-dimension space and two-dimension space, the inference is drawn that there may be a fourth dimension in our space, and that our ignorance of it arises only from the limitation of our perceptive faculties.

These suppositions, however, involve a fatal confusion of mathematical with physical conceptions. Mathematical lines and plane figures do not, like matter, occupy space, and they present no obstruction to the movements of one another. They may freely intersect, or pass through one another, or coincide wholly or in part with one another. If these supposititious beings in one- or two-dimensional space find any obstruction to their movements, it must be because they occupy space, and therefore are really in three-dimension space, however little they extend except in one or two directions. A line or a plane surface can be conceived only with space around it in every direction. The supposition of a one-dimension or a two-dimension space is therefore impossible except as a mathematical abstraction, and furnishes no basis for belief in a fourth dimension.

(2) The straight line, a one-dimension magnitude, ends in points; the square, a two-dimension magnitude, is bounded by straight lines, one-dimension magnitudes; the cube, a three-dimension magnitude, is bounded by squares, two-dimension magnitudes. It is inferred by analogy that three-dimension magnitudes bound four-dimension magnitudes, although the latter are not known by us. Thus the "four-dimensional cube" receives a name, the "tessaract," and is said to be bounded by cubes.

But there is no such analogy as is here assumed. All lines end in points, although some lines, like circular arcs, require two-dimensional space, and others, like a corkscrew curve, three-dimensional. Nor are all two-dimensional figures bounded by straight lines. The bounding lines of circles and ellipses, for example, require two-dimensional space, as much as the figures themselves. Still further, solids like spheres or egg-shaped bodies, are bounded by three-dimension surfaces. There is, therefore, no regular progression which would lead us to suppose the existence of magnitudes bounded by solids. In fact such a supposition is inconceivable. The only possible boundary of a solid is a surface, whatever be the number of the dimensions of space.

(3) In the series of the successive powers of a number, a, a^2, a^3, a^4, . . . a^n, a may be represented graphically by a straight line, of which a denotes the length; a^2, by a square, of which a denotes the length of a side; a^3, by a cube, of which a denotes the length of an edge. It is inferred that if we keep on, there must be a magnitude corresponding with a^4, and so on indefinitely up to a^n. Such magnitudes are incompatible with three-dimension space, and suggest for their possible existence "spaces of higher order."

To those who have some elementary knowledge of analytical geometry, or even of the use of graphs in algebra, the origin of the conception of spaces of higher order may be presented in a different way. As an equation containing two "variables" may be considered as representing the locus of a series of points in a plane, so an equation with three variables is the locus of points in space, referred to three rectangular axes. But since, as shown above, in explaining the word "dimension," three dimensions or co-ordinates fix definitely and exactly the position of a point, equations with more than three variables transcend the scope of our geometry, and require for analogous interpretation spaces of more than three dimensions.

There is no objection to the hypothesis of spaces of a "higher order" as a purely mathematical conception; but this abstract supposition has no bearing on the number of dimensions of actual space as we know it.

(4) If we connect by a straight line the upper extremities of a capital V, we form what is called an isosceles triangle. If we connect the middle point of our line with the bottom point of the V, we have divided the triangle into two triangles which are plainly equal. If we were confined to the two-dimensional surface of which the triangles are a portion, we could never move them about so as to apply one to the other, and prove them equal by coincidence. Not being under this restriction, but being in three-dimension space, we turn one of the triangles a half revolution on one of its sides, and then the two figures may be made to coincide. Now there are many symmetrical solids, for instance, the two hands, which can never be brought into identical shape. We cannot prove the left hand equal to the right by putting on the left the right-hand glove. But if we turn the right hand glove inside out it will fit the left hand. Just as we can prove two-dimensional figures equal by availing ourselves of the possibilities presented by three-dimensional space, it is inferred that in four-dimensional space, not only the glove, but the hand within it, might be turned inside out, and made identical in shape with the other hand. No explanation is offered of the way in which an additional dimension would render such a eversion possible, and if we could admit that it would do this, we are not shown why the actual existence of a fourth dimension follows. Some four-dimension enthusiasts appear to believe that symmetrical forms in organic bodies could not originate without a fourth dimension, but no reason is given for this belief.

The four numbered sections above include virtually all the lines of thought along which the effort is made to substantiate the existence of a fourth dimension. Metaphysical considerations are sometimes added of the uncertainty and possible inaccuracy of our conception of space, but with no suggestion for correcting this inaccuracy, and no argument for the belief in a fourth dimension. Admit that the mind must itself contribute an *a priori* element to all knowledge, and that the truth of things is not limited by the phenomenal apprehension of them; it does not follow that this apprehension is to be assumed without demonstration to be false or incomplete. In an investigation like the present one it is unnecessary to consider whether our conception of the *non ego* is subjective or objective; we must accept the world of matter and of mind in which we live as our perceptions present it to us, and as it is generally conceived. No observation has ever discovered the existence of a fourth dimension in space, and it may safely be said that there is no reason for believing in its existence.

The theory of spaces of a higher order, as developed in section (3) above, is entirely legitimate as an abstract mathematical conception, but furnishes no basis for the supposition of a fourth dimension in our space. It virtually assumes space as we know it to be three-dimensional; yet from a suggestion arising from this theory apparently (for no other origin for the assumption is to be found) the four-dimensionists have made space as we know it a space of the highest order; for the same analogies and inferences on which they depend would lead us to a fifth, a sixth, an nth dimension. A fourth dimension belongs (or rather four dimensions belong) to the theoretical four-dimension space; but mathematics furnishes no basis for ascribing to our space more or fewer than three dimensions.

The confusion of thought of the four-dimensionists characterizes their writings on the subject. The most thorough-going devotee of the fourth dimension asserts: "There is nothing mysterious at all about it . . . From every particle of matter there is a new direction, not connected with any of those which we know, but independent of all the paths we can draw in space, and at right angles to them all." It would seem indisputable that a direction at right angles with all the paths or lines that we can draw in space from any point, would produce lines coinciding with all the lines drawn from the point, and therefore giving no "new direction." But we do not need to be convinced that there is no "direction" from which we are cut off, and in which we cannot direct our perceptions.

The attempted analogies described in section (1) above, are those on which the four-dimensionists chiefly depend, and they rely upon them to show that a fourth dimension would explain how a body may become invisible. They assert that a body would disappear on "entering the fourth dimension." This expression is manifestly unintelligible. Every body extends constantly in all the dimensions of space; we cannot think of it as "entering the dimension" of length, breadth, or of thickness, or of "entering the fourth dimension," if there were one. But the disappearances produced as in section (1) depend wholly on removal from the limited perceptive faculties of the supposed observers; but our normal perceptions are unrestricted in direction, and extend to every point in space, unless cut off by distance or by an interposed physical obstruction. If all the particles of a body moved in the "new direction" of the imaginary fourth dimension, the body would still retain its length, breadth, and thickness, and would still remain within the range of our perceptions.

The assertion is made on the authority of eminent mathematicians, that in space of four dimensions there would be no obstruction to entering or emerging from space shut in on every side, as a tightly shut box or room, and "the fourth dimension" is relied upon to explain supposed mysterious occurrences of such entrance or emergence. The modification of physical laws in spaces of a higher order, those of unusual mathematical ability alone can be expected to understand, and in the special instance in question no explanation is vouchsafed. Until such explanation is given, those who can make no claims to exceptional mathematical talent will be unable to believe it possible, in space of the fourth, or of any order, to extract the contents of an egg, or to pass an object within the egg, and at the same time leave intact the continuous material structure that we call the shell. But whatever may be possible in theoretical spaces of higher order, we need not accept an unintelligible fourth dimension to aid in the explanation of something equally unintelligible.

It may be said in conclusion, that the only "explanation of the fourth dimension" that can reasonably be given, is to say that, in the sense in which the expression is used, the fourth dimension is absolutely non-existent. It could have meaning only to designate the dimension, in addition to the three that we know, belonging to the imaginary mathematical hypothesis of four-dimension space. The "fourth dimension" has no relation to the actual universe in which our sensations and perceptions are exercised, and belongs to that realm of thought to be entered only by the select few, whose exceptional genius includes the development of the mathematical imagination.