THE SEVEN BRIDGES OF KÖNIGSBERG AND OTHER PUZZLES.

BY J. F. SPRINGER.

The ancient and university town of Königsberg is situated on the river Pregel, which here forms an island called Kneiphof. There are seven bridges over the river, five of which connect with the island. In the earlier part of the nineteenth century a discussion arose as to whether it were possible for a person to pass over all the bridges in one continuous trip and without covering the same path twice. In fact, this problem attracted the attention of the celebrated mathematician Euler. In order to understand the question clearly, refer to the map. The start may be made from any point. The problem is really insoluble, try however you will. However, if it be considered allowable to cross the Pregel by the railroad bridge below the town, the problem may readily be solved. Thus, beginning at a point on *D* one passes over the Holz Bridge, then over the Schmiede Bridge to the island, then back to *C* by the Krämer Bridge. One now makes a detour, passing over the Pregel from *C* to *B* by the railroad bridge, then passes to the island by the Grüne Bridge, returns over the Kötte Bridge, goes over the Hohe Bridge from *B* to *D*, and finally completes the journey by crossing the Honig Bridge onto the island. Thus, seven—in fact, eight—bridges have now been crossed and no part of the path has been covered twice.

This type of problem may fittingly be termed a *traveling puzzle*. It is in reality a very ancient kind of thing. Thus, there has come down to us from the time of Pythagoras, who flourished in the sixth century before the present era, a very simple example in the shape of the Pythagorean star, an illustration of which is annexed, Fig. 2. This figure may readily be traced by one continuous line and without duplication of the path.

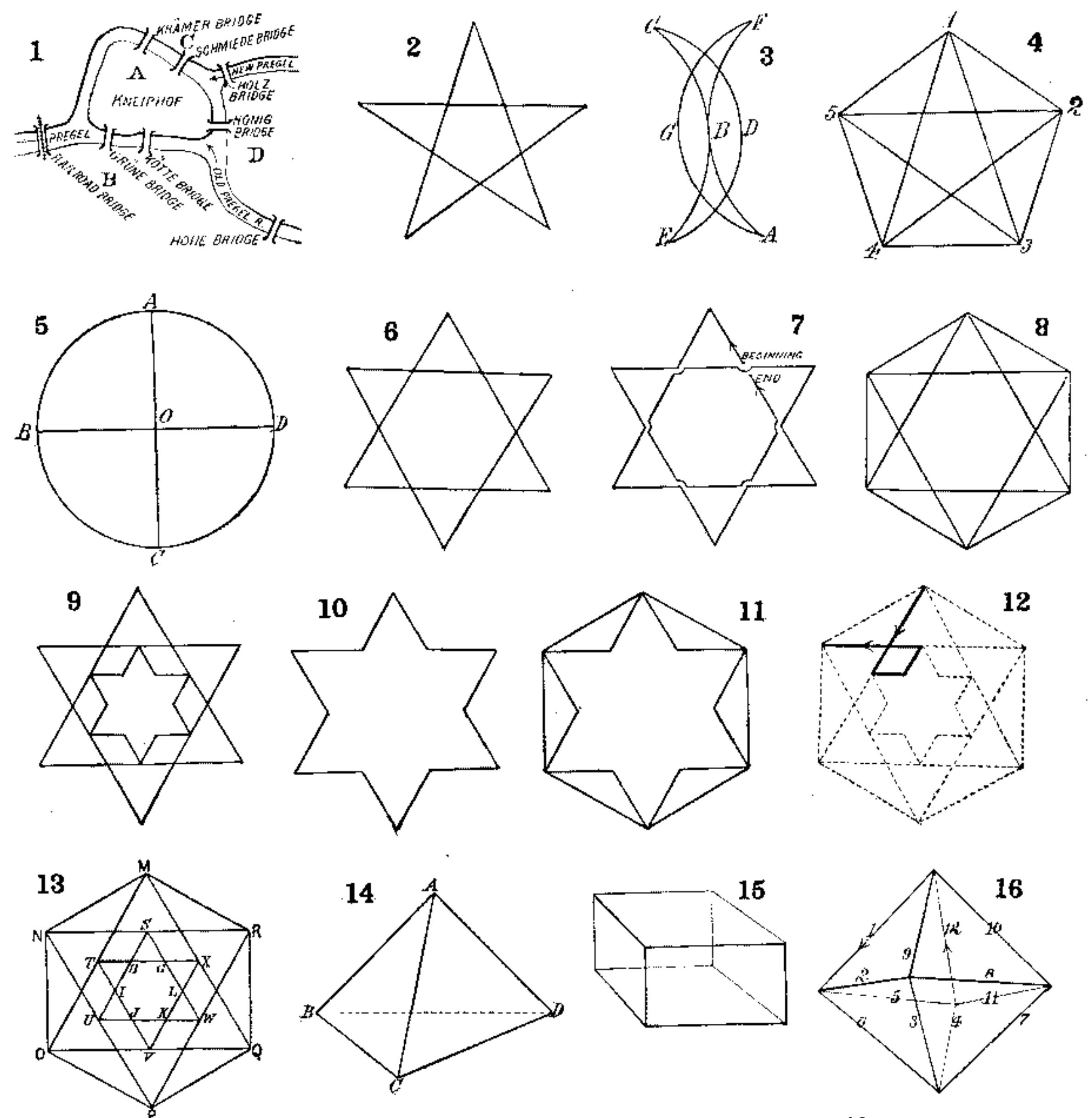
A story is told to the general effect that a disciple of Pythagoras once fell sick at an inn, where he was cared for very kindly by the innkeeper. Instead of getting better, however, he grew worse. At last, with the expectation of dying and being unable to repay his kind host, the Pythagorean asked for a board. When this was brought, he traced out the single-line star. Giving this to the innkeeper, he desired him that it should be displayed outside. Some time after his burial, a stranger happened along. Upon observing the star, he made inquiry, and was informed of the particulars related. He then, in order no doubt to make the story complete, handsomely rewarded the innkeeper for his unselfish care of the unfortunate Pythagorean.

Another figure of the single-line type is that known as Mohammed's signature. This is shown in the annexed drawing, Fig. 3. It is understood to have been drawn by Mohammed upon the sand by a continuous and unrepeated movement of the point of his scimetar. Beginning at *A* and following the course indicated by the letters *ABCDEBFGA*, one may see how it was possible to accomplish this result.

An extension of the Pythagorean star is shown in Fig. 4. This may be solved by following the routes indicated by 1 2 3 4 5 1 4 2 5 3 1, 1 2 3 4 5 1 3 5 2 4 1, and 1 4 3 1 5 4 2 5 3 2 1. In these it will be noticed that two or more exterior sides are taken consecutively. If it be required that this shall not be the case, the problem is perhaps somewhat more difficult. Nevertheless, it is soluble, as may be seen by following out the order indicated by 14315325421.

We must not be deceived by the apparent simplicity of a given case of this type of puzzle. Thus Fig. 5 discloses the very simple figure made by a circumference and two diameters. Try as you will, you cannot cover this figure by a continuous line that nowhere duplicates itself. On the other hand, figures that are apparently very complicated frequently admit of a ready solution. Thus, the six-pointed star shown in Fig. 6 may be quickly solved by the method shown in Fig. 7. To work the puzzle given by Fig. 8—that is, the star of Fig. 6 with the including polygon—observe Fig. 7. This does not in its present form, perhaps, suggest a solution, for the reason that beginning and ending at the point indicated, we have no opportunity to draw the inclosing hexagon, either as a preliminary to starting or as a sequel to finishing. But at the moment when we have arrived at the tip of *any* of the six points of the star we may draw this hexagon, and then continue according to Fig. 7.

Refer now to Fig. 9. This is apparently a very complicated design. There is a very simple solution, however, which Figs. 10, 11, and 12 will assist in developing. It is easy to see how to draw Fig. 10, no matter where we elect to start. If we start at the tip of a point, the including polygon of Fig. 9 may easily be drawn as a preliminary or a sequel (Fig. 11). There is just one thing to see, and that is how the remainder of Fig. 9 may easily be made by forming a kind of loop at each of the inner points, *A, B, C, D, E, F* (Fig. 11). The method of making this loop is indicated in Fig. 12.



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37	62	43	56	35	60	41	50
44	55	36	61	42	49	34	59
63	38	53	46	57	40	51	48
54	45	64	39	52	47	58	33
1	26	15	20	7	32	13	22
16	19	8	25	14	21	6	31
27	2	17	10	29	4	23	12
18	9	28	3	24	11	30	5

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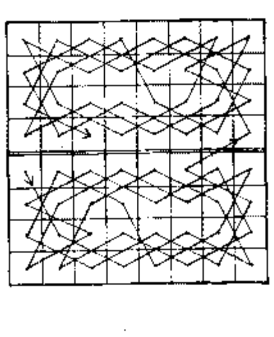

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50	11	24	63	14	37	26	35
23	62	51	12	25	34	15	38
10	49	64	21	40	13	36	27
61	22	9	52	33	28	39	16
48	7	60	1	20	41	54	29
59	4	45	8	53	32	17	42
6	47	2	57	44	19	30	55
3	58	5	46	31	56	43	18

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To draw Fig. 13, we proceed as per Fig. 9, except that the moment of arrival at *any* one of the points *G, H, I, J, K, L* is selected as the time to draw the innermost hexagon. A complete solution is afforded by the course indicated by *M T H S N U I T O V J U P W K V Q X L W R S G* **H I J K L G** *X M* **N O P Q R M.** The heavy letters indicate where the innermost and outermost hexagons are added.

Comparing Figs. 5 and 13, it may seem hard to realize that one puzzle may be worked and the other not. Perhaps some readers may be inclined to think Fig. 5 soluble. An actual solution will of course prove that they are right. In the meantime, the following considerations may prove of interest: There are in all five junction points—*O, A, B, C, D*. If we do not start or end at such a point, we must recede from it for every approach; and conversely, for every recession there must have been a previous approach. Approaches and departures are thus paired off. At a starting point, however, it is possible to have a departure without a previous approach; this would occur when we begin, and only then. Likewise at a finishing point, we may have an approach without a following departure; this would occur at the end, and only then. That is to say, there can not be more than two points (the start and finish) where an odd number of lines join. In Fig. 5 there are *four* such points—*A, B, C, D*. This shows sufficient reason for pronouncing this figure insoluble.

Let us turn now to solid bodies, and look at some of the simpler cases. Take the tetrahedron shown in Fig. 14. It is certainly a matter of indifference at which vertex we begin, so we start at *A*. We have the choice of three beginnings. It is also evidently a matter of indifference which of these we follow, so we pass to *B*. Here again the two possible choices are alike, so we go to *C*. Here the two routes lead to different results—*C A* completing a triangle (*ABC*) and *C D* closing no figure. First we try *CA*. Arrived at *A*, we are compelled to go to *D*. We have now two lines to draw—*D B* and *D C*. We may cover one, but not both. So then we return to *C* and try *C D*. Arrived at *D*, we see that if we go to *B* we shall be unable to go any farther. So then we go to *A*, and thus are forced to *C*. Here we stop, with *D B* undrawn. Referring, however, to the discussion of Fig. 5, we observe that the tetrahedron comes under the head of the impossible figures, as there are four points where an odd number of lines join, viz., *A, B, C, D*.

Fig. 15 is likewise an insoluble case, having eight points where three lines join. Fig. 16 is an apparent advance in complication. But we observe that all six vertices are junction points for an even number of lines. It is, in fact, a soluble case, as may be seen by following the course indicated by the numerals.

Another variety of this same general class of puzzle is the problem which requires the knight to start from a position on the chessboard and cover the whole board by a continuous series of moves, no position to be taken more than once. A convenient way of trying this puzzle is to rule with a sharp instrument on a slate the sixty-four squares of the chessboard. Wherever you elect to start the knight, you mark 1. His next position you mark 2, and so on. The slate enables false starts and errors to be readily corrected. This kind of puzzle has attracted a good deal of attention, and has received a multitude of solutions. Thus we may instance the solution given in Fig. 17. Here the lower half of the board is covered before any beginning is made with the upper half. The two halves are precisely symmetrical with each other, as may be seen by referring to Fig. 18, where the path of the knight is indicated by a continuous line. This division of the solution into two duplicates is not necessary, but is an added refinement. In one sense it simplifies matters, as we have but half the board actually to solve. We are restricted, however, as to the point of termination. Thus in the present example, the point of beginning, 1, having been determined, the point 33—the beginning of the second half—is thereby fixed, so 32 must come where it is at present or must be at position 6. Fig. 19 is an illustration of a solution where the resulting arrangement of figures has some of the properties of a magic square. Thus every column and every horizontal line sums up 260. If the diagonals each totaled the same number, 260, then the whole would form a perfect magic square.

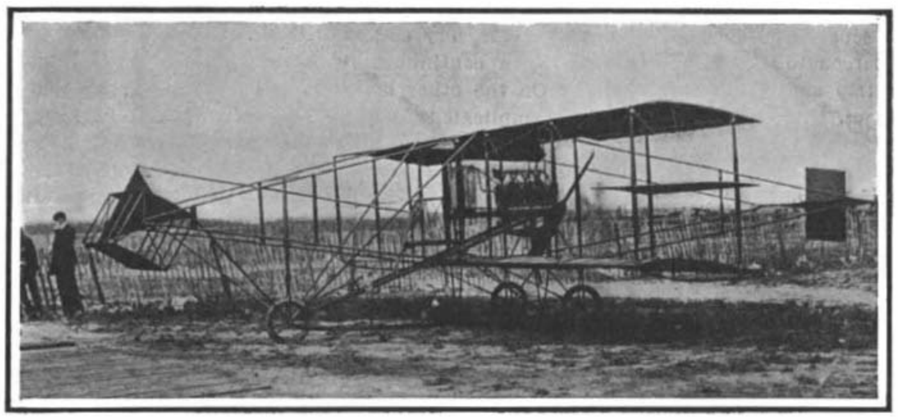
TERMINATION OF THE RHEIMS AVIATION MEETING.

THE WINNING OF THE INTERNATIONAL TROPHY.

As briefly noted in our last issue, Glenn H. Curtiss won the Bennett International Aviation Trophy on August 28th at Rheims. This trophy—a beautiful model of a Wright biplane held aloft by a female figure—was contested for the first time on the date above mentioned, France being represented by two monoplanes—a Bleriot and an Antoinette—and one Wright biplane, and America by one tiny biplane with a powerful 8-cylinder motor. The real race was between Curtiss and Bleriot, the champions of the biplane and the monoplane types of flying machines respectively; and that the former accurately sized up his rival soon after he reached France is shown by the fascimile reproduction of the postal which he at that time sent our Aeronautic Editor.

The morning of August 28th was mild, calm, and hazy at Rheims. As the weather conditions were so favorable, Mr. Curtiss brought out his machine a few minutes after 10, and immediately started off on a preliminary round of the course. Despite the fact that he made rather wide turns and that the aeroplane pitched considerably, the time of the round was but 7 minutes, 55 1/5 seconds—a decided improvement over Curtiss's former fastest round of 8:09 1/5, and 9 1/5 seconds less than Bleriot's fastest lap. Mr. Curtiss decided to try for the trophy at once. His small gasoline tank was refilled, more water was put in the radiator, and, after signing the official paper, he quickly rose for the second time. After circling around once in front of the grand stand, he crossed the line at full speed. The aeroplane still pitched perceptibly, and the turns were, with the exception of the very last one, all rather wide; but nevertheless both rounds were made in record time, the second one being 4 1/5 seconds faster than the first and 2 seconds faster than the time in the trial flight. The times of the rounds were 7:57 2/5 and 7:53 1/5, the total being 15 minutes, 50 3/5 seconds, which corresponds to an average speed of 47.04 miles an hour.

The 4 1/5 seconds gain in time on the second round, Mr. Curtiss attributed to a slight change in the mixture which he effected by turning a small wheel he had conveniently at hand. He ran the engine at its fastest speed all the time, but during the second lap thought that it started missing explosions on one cylinder, so he made a slight adjustment. The pitching of the machine seems to have been due to the fast speed at which it was being driven in conjunction with the sensitiveness of the horizontal rudder control. When the machine would pitch downward and Mr. Curtiss would turn it upward again, the shock was so great that he said it was like striking a bump when riding in a fast automobile.



The Curtiss biplane which won the International Aviation Trophy.

This machine made one circuit of the course at the rate of 47.73 miles an hour. It traveled 18.63 miles in 15 minutes 50¾ seconds (47.04 miles an hour) in winning the International Trophy.

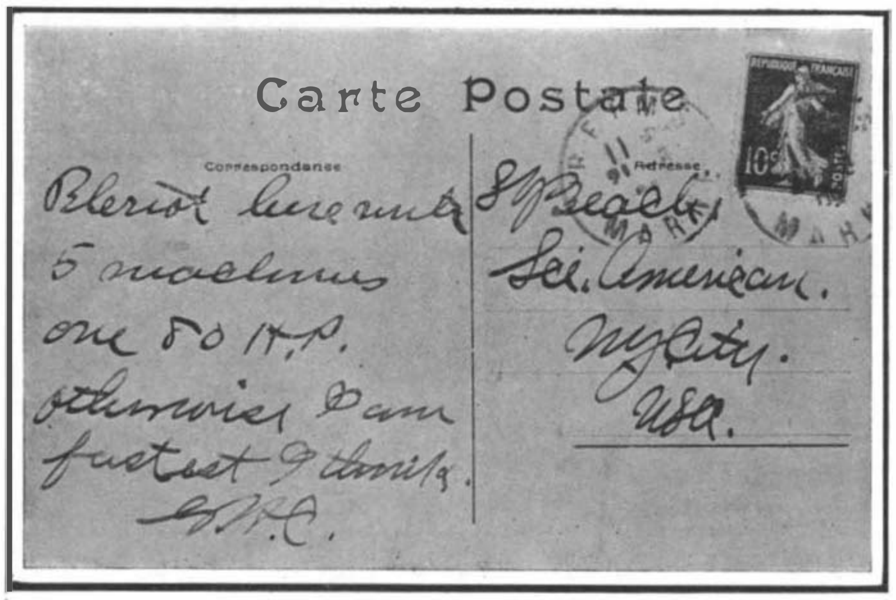
The other main event of the day—the Prix du Tour de Piste, or 10-kilometer speed test for one circuit of the course—did not bring out any contestants, since the two events could not be run together, and since competitors were obliged to state for which prize they were competing. But one trial was allowed for the International Trophy, also. After Curtiss's excellent flight, no other machines were brought out till about noon, when M. Bleriot made a slow round with his 80-horse-power "No. 22" monoplane. About 2 P. M. he tried another propeller, but only succeeded in making a round in 8:14 1/5. An hour later he had changed the 2-bladed propeller for a 4-bladed one. He attempted to make a round, but was obliged to descend before completing it. After working at the monoplane where it alighted for over an hour with the aid of several mechanics, he at length flew back to his shed. As it was now almost 5 P. M., and as no start was allowed after 5:30, he made hurried preparations for the test. It was 5:10 before the start was made. The monoplane flew splendidly without any rolling or pitching. The time of the first round was but 7:47 4/5, which was 5 2/5 seconds faster than Curtiss's second lap. If Bleriot could do as well in the second round, he would be the winner. There was intense excitement among the spectators at the grand stand. The machine finally rounded the last pylon. The timers called the seconds remaining before his time would be up, but it was 5 3/5 seconds over Curtiss's 15:50 3/5 before the indomitable Frenchman crossed the line. After conquering the Channel he had finally been defeated for lack of speed. Nevertheless, the performances of his and Latham's monoplanes remained unsurpassed for stability, even in strong winds. The latter started in the cup competition just as Bleriot was finishing. He flew at a great height—about 150 feet—and covered the course in 17 minutes, 32 seconds, thus securing third place. Lefebvre, the third French representative, with a Wright biplane fitted with a 40-horse-power motor, was fourth in 20:47. Mr. Cockburn, who represented England with his Farman biplane, got half way around the course when the end of one plane struck a standing shock of corn, whirling the aeroplane around and bringing it to the ground. Latham met with a similar mishap afterward when carrying M. Sariano as a passenger in the passenger-carrying competition. This was won by M. Farman, who, after making a round with one passenger in 9:53 4/5, afterward carried two around the course in 10:39 2/5, or at a speed of 34.96 miles an hour. The total live weight lifted by his machine was in the neighborhood of 450 pounds. A Wright biplane carried Franz Reichel around the course in 11:05 4/5. Farman's biplane was the only machine that succeded in carrying three people. Bleriot's "No. 12" monoplane, however, was the first aeroplane to accomplish this feat, which it did at Douai last June, when a total weight of 1,234 pounds was carried at about 30 miles an hour with a 30-horse-power motor. Farman's biplane had a 50-horse-power Gnome revolving-cylinder motor. This engine was fully described in Supplement No. 1729.

The Bleriot monoplanes in front of their sheds.

The No. 22 machine was fitted with an 80-horse-power, 8-cylinder motor. It made the fastest round of the course at the rate of 47.78 miles an hour.

In addition to winning the International Trophy Mr. Curtiss, the following day, carried off the first prize ($2,000) in the 30-kilometer speed contest, known as the Prix de la Vitesse. His first attempt was made early in the afternoon. The three rounds of the course were made in 24 minutes 15 1/5 seconds. Believing that Latham had made better time, he made another attempt. This time he made very short turns and drove his machine at even greater speed. The three rounds were made in 7:49 2/5, 7:48 2/5, and 7:51 1/5, the total time for the three laps being 23 minutes and 29 seconds, or a speed of 47.6 miles an hour. The second lap was made at a speed of 47.73 miles an hour, which was the fastest time for the course by any machine, with but one exception. Because Mr. Curtiss did not start in this contest on the first day of the meeting, he was penalized 1/20th of his actual time, so that his official figures were 29 minutes and 49 seconds. Latham made another attempt to better his previous record, but in this he was unsuccessful.

Bleriot started about 10 o'clock with the intention of making another trial in this competition. He crossed the line and made the first turn at a rapid rate, flying at a low elevation. He finally disappeared from



Postal card showing how Curtiss sized up his opponents.

THE RHEIMS AVIATION MEETING.