Correspondence.

PROPERTIES OF NUMBERS.

To the Editor of the SCIENTIFIC AMERICAN:

This article is suggested by the two articles on numbers published in the SCIENTIFIC AMERICAN, March 28 and November 21, 1908.

The pleasure which the mathematician experiences on the discovery of a new theorem has a character peculiarly its own. The schoolboy experiences the feeling when he gets the correct answer to a particularly hard problem. The gratification is undoubtedly due to the absolute sureness of the result. Naturally, there is often a corresponding pain, the disillusionment of the discoverer on learning that his discovery is hoary with age.

The formula for constructing two square numbers whose sum is a square, i. e. $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2\ xy)^2$, given by your correspondent in the issue of November 21, was first given 1700 years ago by Diophantos of Alexandria. The formula is made more general by multiplying throughout by k^2. Numerous mathematicians have given this and allied subjects attention. Mention may be made particularly of Fermat and of Gauss, who was probably the greatest mathematician in history. Fermat proposed the now famous theorem, to prove $x^n + y^n = z^n$, is not possible in integers for any integral values of n greater than 2. This has been proved for $n = 3$, $n = 4$, and for all values of n up to 100. The general proof awaits solution, and the finder of the solution will receive the prize of 100,000 marks ($25,000), left by M. P. Wolfskehl. Profound work along this line has been done by Kummer and Hilbert; it is quite certain that any solution must be based on the work of these scholars. It is also quite certain, as L'Enseignement Mathematique remarks, that the desire to gain 100,000 marks is evidently much more common than the comprehension of the fundamental theorems of modern mathematics which are necessary for the solution of this problem. As in the case of the squaring of the circle, the duplication of the cube, and the trisection of an angle (by straight line and circle) many false prophets will arise to demonstrate. These three problems have all been shown to be not solvable with ruler and compass.

Chrystal's Algebra (1900) gives as an exercise (p. 534, ex. 12) the problem that the cube of every rational number is the difference of the squares of two rational numbers. The problem itself is not original with Chrystal, but is much older.

That $x^3 - x$ is divisible by 3, $x^5 - x$ by 5, $x^7 - x$ by 7, $x^{11} - x$ by 11, $x^{13} - x$ by 13, depends on the well-known Fermat's theorem, published in 1670, that x^{m-1} is congruent to 1 modulus m when m is a prime number, x being any number not divisible by m. This means simply that when x^{m-1} is divided by m the remainder will be 1. The proof is elementary, and is given in any Theory of Numbers as well as on page 550 of the second volume of Chrystal's Algebra. Chrystal states explicitly the theorem that $x^m - x$ is divisible by m. By Fermat's Theorem it follows that $x^{13} - x$ is divisible not only by 13, but also by 2, 3, 5, and 7, since $x^{13} - x$ contains besides the factor x the factors $x - 1$, $x^2 - 1$, $x^4 - 1$, and $x^6 - 1$. Consequently, $x^{13} - x$ is divisible not only by 910, as given by your correspondent, but even by 2730.

To actually calculate $12^{13} - 12$, in order to prove the result divisible by 13, would be termed a mathematical barbarity. $12^{13} - 12 = 12\ (12^{12} - 1)$.

$12 = 13 - 1$. $12^{12} = (13 - 1)^{12}$, which when expanded by the binomial theorem shows that every term except the last contains the factor 13, and the last term is $+ 1$. Subtracting 1, the expression is divisible by 13. Similarly, to prove $7^{13} - 7$ divisible by 13, we write $7^{13} - 7 = 7\ (7^{12} - 1)$. $7^{12} = (13 - 6)^{12}$, which when expanded by the binomial theorem has every term except 6^{12} divisible by 13.

$6^{12} = (6^2)^6 = (36)^6 = (39 - 3)^6$, of which all terms except 3^6 are divisible by 13. $3^6 = (27)^2 = (26 + 1)^2$, which expression gives the remainder 1 when divided by 13. Therefore $7^{12} - 1$ is divisible by 13. Of course, this work is entirely unnecessary, as the results are proved by the general theorem.

$1297^{1901} - 1297$ is divisible by 1901, by the theorem (since 1901 is a prime number), and it would take a goodly portion of a man's life to verify the fact by computation. 1297^{1901} would be written with 6,118 digits; written out, it would take up about a column of the SCIENTIFIC AMERICAN.

The method given for constructing a right angle by using cords or boards of lengths 3, 4, and 5 feet—or 12, 16, and 20 feet—is hinted at in Egyptian records that are 4,000 years old. Heron of Alexandria, writing about 2,000 years ago, gives quite a full explanation of the matter.

The ordinary proof by nines is more than a thousand years old, as it is given by Alchwarizmi, an Arabic writer who dates back to 825 A. D. A proof by elevens was given by the Arab Abu Bekr Muhammed ibu Alkarchi about 1000 A. D., while the particular form mentioned of calculating the difference between the sums of the even and odd digits is found in many European arithmetics from 1750 on down to 1850.

That 142,857 when multiplied by 1, 2, 3, 4, 5, and 6 gives the same succession of digits rearranged in cyclical order (i. e., as though these six numbers were written around a circle) is explained by the fact that 142,857 is the repetend of the repeating decimal of $\frac{1}{7}$. $\frac{1}{7} = .\dot{1}4285\dot{7}$. Since in dividing 1 by 7 there appear all six different remainders from 1 to 6, it follows that when 2 is divided by 7, the same six remainders will appear beginning at a different point.

$1 \times 142{,}857 = 142{,}857$
$2 \times 142{,}857 = 285{,}714$
$3 \times 142{,}857 = 428{,}571$
$4 \times 142{,}857 = 571{,}428$
$5 \times 142{,}857 = 714{,}285$
$6 \times 142{,}857 = 857{,}142$
$7 \times 142{,}857 = 999{,}999$

Using these seven arithmetical facts, one can multiply offhand 142,857 by any number; e. g., to multiply by 233, divide 233 mentally by 7, giving 33 3/7. Write the 33 first, followed by the succession for 3/7; finally subtracting 33. This gives 33/4285.38. Since $7 \times 142{,}857$ gives 999,999, or 1,000,000 — 1; $33 \times 7 \times 142{,}857$ gives 33,000,000 — 33. It is evident that the seven facts need not be written down; the repeating decimal for 3/7 begins with 4 and has same succession of digits in same order as the decimal for 1/7.

The sequence given $.\dot{0}52,631,578,947,368,42\dot{1}$ repeating gives 1/19; as there are 18 places in the repetend, all eighteen remainders have appeared. Consequently, the same sequence of digits in cyclical order is obtained when you multiply this number by any number from 1 to 18. A similar rule holds for multiplying mentally this number by any number; e. g., to multiply by 254, divide 254 by 19, giving 13 7/19. This gives the product as 13/368,421,052,631,578,934. The first 13 is the integral part of $\frac{254}{19}$; the second part is the repeating part of the decimal for 7/19 with the exception of the last two places, which are obtained by subtracting 13 from the final 47.

$19 \times 0.052{,}631{,}578{,}947{,}368{,}421$ gives 999,999,999,999,999,999, or 1,000,000,000,000,000,000 — 1. This fact combined with the cyclical permutations when multiplied by numbers from 1 to 18 gives the explanation of the method.

Tradition holds that the Hindus, to whom we are indebted for our system of numbers misnamed the Arabic, were the first people to occupy themselves with magic squares. On a copper engraving of Melancolia made by Albrecht Dürer about 1500 there is depicted a small magic square. The magic square given by your correspondent is not what is termed a regular magic square, as this is supposed to use all the integers from 1 up to 18^2. About 1835 a three-volumed work appeared on the subject of magic squares, and within a month an American firm (Open Court) has brought out a small work on the same subject.

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GOVERNMENTAL INCOME AND OUTGO.

On March 16th President Taft transmitted to Congress a message pointing out the necessity for a revision of the tariff. On March 17th Chairman Payne of the Ways and Means Committee introduced the new tariff bill, on which his committee had been working steadily for four months. It is not within the province of the SCIENTIFIC AMERICAN to comment on the merits or imperfections of this bill, which will undoubtedly be more or less modified in the legislative alembic, but it is interesting to know what that highly complex organization of highly complex units actually costs, and where the revenue actually comes from.

The Division of Bookkeeping and Warrants of the Treasury Department states that the receipts derived from Customs, Internal Revenue, sales of Public Lands, and miscellaneous sources was $601,126,118 for the fiscal year ending June 30th, 1908, while in the same period disbursements exclusive of the principal of the public debt were $659,196,319. It is this deficit which the new tariff will wipe out, as well as give increased revenue for public works, increased protection, etc.

For purposes of comparison we have represented the receipts and expenditures in graphical form, using the revolving door as a means to this end. Customs furnish the largest item, amounting to $286,113,000; Internal Revenue follows with $251,711,000, then come the other larger sources of income, each man being shown of a size normal to the amount of money he is supposed to be carrying.

The "Outgo," represented on the other side of the door, shows the soldier, the sailor, the postman, the veteran, etc., of the proper size. The amounts are so clearly shown that they do not need recapitulation here. Smaller items are not shown, as then the pictures would be misleading.

It is needless to say that the appropriations and expenditures of the government are increasing annually, but the revenues from imports have recently shown substantial gains; and should these continue, as there is every reason to believe that they will, the Treasury will be in good condition to await the approaching readjustment of the tariff.

The Taylor-White Steel Patents Held Invalid.

The two patents granted to F. W. Taylor and M. White (668,369 and 668,270), which apply substantially to all steels for cutting tools in the composition of which chromium and tungsten or molybdenum appear, and to all temperatures employed in treating such steel for machine-tool purposes in excess of 172 deg. F., have been declared invalid in the suit brought by the Bethlehem Steel Company against the Niles-Bement-Pond Company for infringement.

Taylor and White invented no new composition of steel. Their patents cover simply a process for the treatment of steel of certain limited compositions. Taylor and White claimed to have discovered that, when air-hardening steels are made with certain constituents in ascertained proportions, the deterioration that ordinarily results at temperatures above a cherry red, prevails only from 1,550 deg. to 1,700 deg. F. (called the "breaking point") and up to a temperature at which the steel softens or crumbles when touched with a rod (approximately 1,900 deg. to 2,000 deg. F.), the efficiency of tools of such special steels—that is to say, their cutting speed and also their uniformity in efficiency—is greatly increased, and largely so in proportion to the degree of heat to which they are raised.

The decision of the court lays great stress on the alleged "breaking-down point" between 1,550 and 1,725 deg., mentioned in the patent. "If such breaking-down point did not exist, or did not exist between the degrees of temperature named, the patentees made no discovery and no invention; or, again, if workers in the art were accustomed to temper their steel by the application of more or less heat, according to its composition, and in its treatment applied temperatures exceeding 1,725 deg., the higher limit of the alleged breaking-down point, the patent must likewise fail."

That the patentees were wrong in their claims followed from tests made in the presence of representatives of both parties to the suit. The result was to show that a heat of 1,500 deg., regarded by the patent as the highest point of efficiency in the prior art, was, indeed, the lowest point of efficiency; that from 1,550 deg. to 1,600 deg. the same degree of efficiency, or rather of inefficiency, was substantially maintained; and that from and after a temperature of about 1,600 deg., and not of 1,725 deg., as called for by the patent, marked improvement was shown. In short, every material assertion of the patent bearing upon the point in question was disproved.

The court held that "it would seem that the prior art need not, and ought not, to be strictly limited to what was done in making metal-cutting tools of the precise character indicated in the patent. The questions involved, broadly considered, have to do with the tempering of steel, the use to which the steel might subsequently be put being relatively unimportant. . . .

"A reasonable application of this principle would seem to broaden the prior art, for if it were customary to temper steel in analogous arts by the application of very high temperatures, equaling and even exceeding those of the patents, and this without injury, then the application of the same treatment, for a like purpose, to steels for metal-cutting tools would not necessarily involve invention. . . .

"The testimony shows that it would have been impossible to make an efficient cutting tool out of the chrome-tungsten steel in vogue for some years prior to the patent in suit at a temperature below those within the patent in suit. Such steels not only required, but, in fact, were given, a much higher temperature than that given to the old carbon and cast steel, and a much higher heat treatment than 1,725 deg. named in the patent as the highest of the temperature defining the breaking-down point. The art developed continuously along this line as the carbon constituent was reduced and the toughening elements were introduced into the composition of steel.

"No satisfactory basis appears in the record for the assertion that the patents in suit led up to or were the means of producing or introducing the high-speed steels. On the contrary, such steels were developed normally along lines laid down and recognized prior to these patents. The process of their development has always been gradual, but at the same time consistent and in a single direction, and may well be characterized as one of degree, and the same may be said of their treatment.

"If the composition of steel were always uniform, the best heat treatment for that particular kind of steel, once ascertained, could safely be followed. But inasmuch as the compositions of steel are not uniform, but variable, and frequently unknown, it has always been more or less a matter of experiment to ascertain the degree of heat requisite for their proper treatment, and it is this experimental practice to ascertain what after all was merely a matter of degree that precludes all possibility of invention in the patents."

The Current Supplement.

A new automobile tilting truck recently constructed for the city of Cologne is the subject of the illustrated article that opens the current SUPPLEMENT, No. 1734. Alcohol as a motor fuel is once more discussed. Day Allen Willey tells how smokeless powder is made, and explains how some ship explosions have occurred. Prof. Reginald Fessenden concludes his masterly treatise on wireless telephony. O. Froehlich describes his new process for refining copper. The wonderful engineering feat of connecting the Simplon and Loetschberg tunnels is described by the Paris correspondent of the SCIENTIFIC AMERICAN. Sir Oliver Lodge writes on the ether of space. A very exhaustive description of the Walschaert valve gear is furnished by C. O. Rogers. The Optics of Skulking and Scouting is a fascinating military subject well handled by W. R. Gilbert. Deslandres' investigations of solar electric phenomena and their relation to terrestrial magnetic perturbations are summarized.

It is announced that the French mints are about to coin for the first time 25- and 10-centime pieces (fractional currency) made of aluminium to take the place of the old copper coins, which are to be withdrawn from currency.